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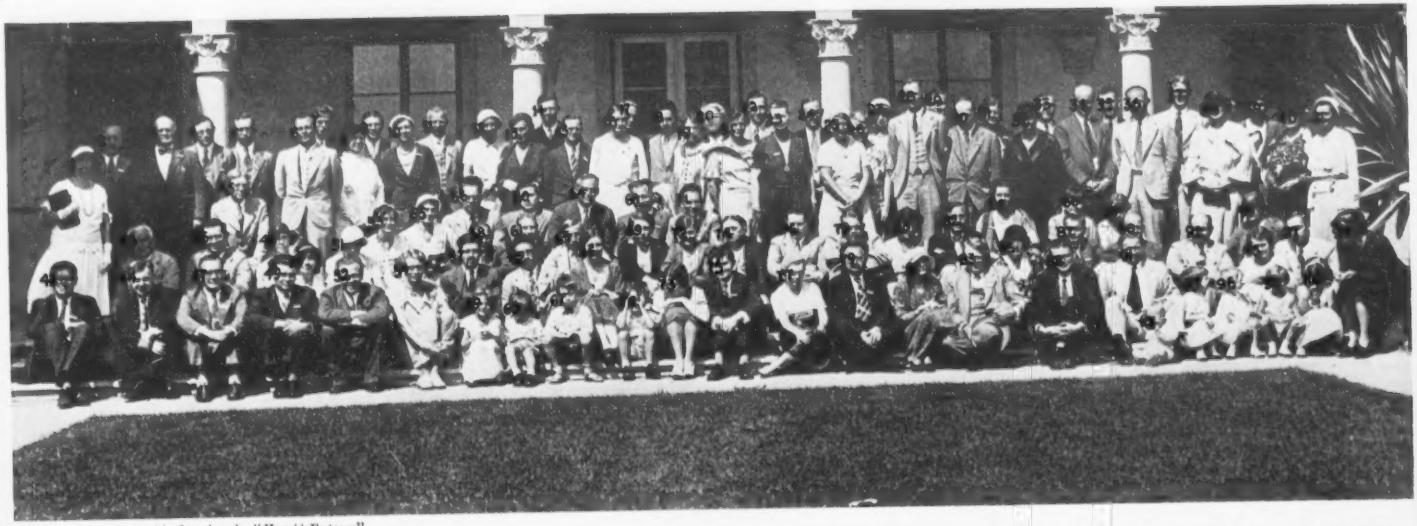

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SUMMER MEETINGS—UNIVERSITY OF CALIFORNIA AT LOS ANGELES. AMERICAN MATHEMATICAL SOCIETY AND MATHEMATICAL ASSOCIATION OF AMERICA. GROUP PICTURE, MYRA HERSHEY HALL, AUGUST 31, 1932

1. Mrs. James McGiffert; 2. E. L. Mickelson; 3. H. B. Leonard; 4. Harry Bateman; 5. C. M. Jensen; 6. M. A. Basoco; 7. R. S. Martin; 8. Myrtie Collier; 9. Edwin Purcell; 10. Mildred Frazier; 11. G. H. Hunt; 12. M. J. Weiss; 13. Jewell Hughes; 14. H. M. Bacon; 15. Hobart Bushey; 16. Beatrice Aitchison; 17. E. T. Bell; 18. H. P. Robertson; 19. M. M. Beenken; 20. G. E. F. Sherwood; 21. Mrs. C. F. Luther; 22. C. F. Luther; 23. C. E. Smith; 24. Raymond Garver; 25. Mrs. Raymond Garver; 26. Mrs. F. W. Owens; 27. H. E. Glazier; 28. W. E. Milne; 29. G. T. Whyburn; 30. J. F. Ritt; 31. Mrs. G. T. Whyburn; 32. Mrs. J. F. Ritt; 33. L. L. Smail; 34. M. H. Ingraham; 35. W. H. McEwen; 36. W. L. Hart; 37. E. R. Hedrick; 38. Mrs. J. P. Ballantine; 39. Edith Hedrick; 40. Mrs. E. R. Hedrick; 41. Lois Griffiths; 42. E. J. Hills; 43. James McGiffert; 44. H. L. Transtrom; 45. F. L. Griffin; 46. L. E. Ward; 47. A. A. Shaw; 48. Mabel G. Whiting; 49. Clifford Bell; 50. Mary E. Schofield; 51. Mrs. M. H. Ingraham; 52. Morgan Ward; 53. Esther Reese; 54. Mrs. Morgan Ward; 55. Mrs. W. E. Mason; 56. W. E. Mason; 57. T. S. Peterson; 58. P. H. Daus; 59. Julia Ballantine; 60. Jack Levine; 61. G. G. Entz; 62. C. G. Jaeger; 63. Charles Ballantine; 64. Dunham Jackson; 65. Glenn James; 66. Mrs. L. E. Ward; 67. Lewis Ward; 68. G. A. Williams; 69. Ruth Mason; 70. Frederick Ballantine; 71. A. L. Buckman; 72. L. P. Copeland; 73. Mrs. A. D. Michal; 74. C. L. Bacon; 75. A. D. Michal; 76. A. E. Ross; 77. Mrs. A. E. Ross; 78. D. H. Lehmer; 79. N. A. Court; 80. Mrs. D. H. Lehmer; 81. Mrs. D. N. Lehmer; 82. Tibor Rado; 83. D. N. Lehmer; 84. Sylvia Parker; 85. Mrs. W. A. Bratton; 86. Mrs. E. L. Mickelson; 87. J. W. Hurst; 88. Imanuel Sperner; 89. Mrs. W. H. McEwen; 90. W. A. Bratton; 91. F. W. Owens; 92. J. P. Ballantine; 93. Helen Rieger; 94. Mrs. M. I. Logsdon; 95. Mrs. W. M. Whyburn; 96. Marjorie Hedrick; 97. W. M. Whyburn; 98. Willa Marie Whyburn; 99. Mrs. W. M. Rieger; 100. Winifred Ingraham; 101. Thomas Aitcheson.

THE SUMMER MEETING IN LOS ANGELES

The thirty-eighth summer meeting and sixteenth colloquium of the Society were held at the University of California at Los Angeles from Tuesday to Friday, August 30–September 2, 1932. The meeting was preceded by that of the Mathematical Association of America. About two hundred persons attended the meeting, among whom were the following eighty-four members of the Society:

Beatrice Aitchison, O. W. Albert, L. D. Ames, C. L. Bacon, H. M. Bacon, C. R. Ballantine, J. P. Ballantine, M. A. Basoco, Harry Bateman, M. M. Beenken, Clifford Bell, E. T. Bell, B. A. Bernstein, F. J. Brand, W. A. Bratton, A. L. Buckman, J. H. Bushey, W. B. Carver, Myrtie Collier, L. P. Copeland, N. A. Court, P. H. Daus, C. H. Dix, G. G. Entz, H. J. Ettlinger, Raymond Garver, H. E. Glazier, F. L. Griffin, L. W. Griffiths, W. L. Hart, E. R. Hedrick, E. J. Hills, J. C. Hughes, G. H. Hunt, J. W. Hurst, C. A. Hutchinson, M. H. Ingraham, Dunham Jackson, C. G. Jaeger, Glenn James, C. M. Jensen, D. H. Lehmer, D. N. Lehmer, H. B. Leonard, Jack Levine, M. I. Logsdon, C. F. Luther, W. H. McEwen, James McGiffert, W. A. Manning, R. G. Mason, W. E. Mason, A. D. Michal, E. L. Mickelson, W. E. Milne, C. V. Newsom, F. W. Owens, H. B. Owens, T. S. Peterson, E. J. Purcell, Tibor Radó, J. F. Ritt, H. P. Robertson, A. E. Ross, I. J. Schoenberg, A. A. Shaw, G. E. F. Sherwood, L. L. Smail, C. E. Smith, Gertrude Smith, D. V. Steed, F. C. Touton, S. E. Urner, H. C. Van Buskirk, H. S. Vandiver, L. E. Ward, Morgan Ward, L. E. Wear, M. J. Weiss, M. G. Whiting, G. T. Whyburn, W. M. Whyburn, C. L. E. Wolfe, E. R. Worthington.

The colloquium lectures, on *Differential equations from the algebraic standpoint*, were delivered on Tuesday afternoon and Wednesday, Thursday, and Friday mornings. These lectures have already appeared in book form, as volume 14 of the Colloquium Publications of this Society.

By invitation of the Committee on Program, Professors D. N. Lehmer and Tibor Radó gave, on Wednesday and Thursday mornings respectively, addresses entitled *The continued fraction representing cubic and higher irrationalities* and *Recent work in the problem of Plateau*.

Tuesday noon those attending the meetings were the guests of Provost E. C. Moore of the University of California at Los Angeles at a delightful luncheon in Kerckhoff Hall. On Tuesday afternoon the ladies of the Department of Mathematics served tea in Kerckhoff Hall.

On Wednesday evening a joint dinner of the Society and the Mathematical Association of America was held at the Beverly Hills Hotel. Professor E. T. Bell acted as toastmaster and called upon Professor E. R. Hedrick who welcomed the guests to California. He was followed by Professor Jewell Hughes, who spoke with pleasant humor. Professor M. H. Ingraham talked about the financial situation and the membership statistics of the Society. Professor W. L. Hart gave the concluding talk of the evening.

Thursday afternoon and evening were devoted to an excursion to the California Institute of Technology, and to the Mount Wilson Observatory. Lunch was served in the Athenaeum at the Institute, and immediately afterward members were shown through its physical laboratories, being given an exhibit of artificial lightning at the High-Tension Laboratory. After dinner at the hotel on the top of Mount Wilson members were shown the hundred-inch telescope of the Mount Wilson Observatory and had an opportunity to look through the sixty-inch instrument.

After lunch on Wednesday a group photograph of those in attendance was taken.

Most of the visitors were housed in Mira Hershey Hall, and meals were served in its dining room. This building also served as social center for the meeting. The arrangements made by Professor Hedrick and the various local committees for the meetings and for the pleasure of the members while in attendance were splendidly planned and executed. All present heartily concurred with Professor Jackson when he called for a rising vote of thanks to all those who had helped in making the meetings a success.

The Council met on the evening of August 30 and various actions taken at this time were reported to the Society.

The following thirteen persons were elected to membership in the Society:

Mr. James Andrew Clarkson, Brown University;
Mr. Paul Eberhart, Brown University;
Dr. Carl H. Fischer, University of Minnesota;
Professor Lofton Leroy Garner, University of North Carolina;
Dr. Ralph Duncan James, California Institute of Technology;
Mr. Holbrook Mann MacNeille, Swarthmore College;
Professor Kenneth Sinclair Purdie, Virginia Military Institute;
Dr. Selby Lemley Robinson, Princeton University;

Mr. Myron Frederick Rosskopf, Brown University;
Mr. Naphtaly Herman Schmellner, West Virginia University;
Dr. Walter George Warnock, Fort Hays Kansas State College;
Miss Frances H. Wiancko, University of Chicago;

As nominee of Yale University on Sustaining Membership:

Miss Miriam F. Becker, Hunter College.

In accordance with reciprocity agreements the following have joined the Society:

Professor Gino Fano, University of Turin;
Professor Harald Geppert, University of Giessen.

The following twenty persons have been elected by mail vote of the Council since its meeting in April:

Mr. Alfred L. Buckman, University of California;
Mr. Franklin Alfred Butter, Jr., Stanford University;
Mr. Livingston Hunter Chambers, West Virginia University;
Miss Eva Lou Culbreth, Columbia University;
Mr. Jack Gordon Deutsch, New York, New York;
Mr. Paul Samuel Donchian, Hartford, Connecticut;
Miss Margaret Gurney, Brown University;
Mr. Eugene Harold Hanson, Ohio State University;
Dr. Miles C. Hartley, University of Illinois;
Professor May Naramore Harwood, Syracuse University;
Mr. Max Alfred Heaslet, Stanford University;
Professor Vinton Asbury Hoyle, University of North Carolina;
Mr. Isidore N. Kagno, Bronx, New York;
Professor Chosaburo Kato, Denison University;
Dr. Ruth Glidden Mason, Berkeley, California;
Mr. Walter Prenowitz, Brooklyn College;
Dr. Helen Gertrude Russell, Wellesley College;
Mr. Herbert Benjamin Weaver, Chester, Pennsylvania;
Mr. Merritt S. Webster, University of Pennsylvania.

The following persons were reported as appointed to represent the Society: at the Centennial Celebration of the founding of the University of Richmond, Richmond, Va., May 8–10, 1932, Professor J. J. Luck; at the Centennial Celebration of the founding of Gettysburg College, Gettysburg, Pa., May 25–30, 1932, Professor F. W. Owens; on the Editorial Board of the American Year Book for a period of three years beginning with 1932, Professor Tomlinson Fort; at the inauguration of William A.

Boylan as President of Brooklyn College, Brooklyn, N. Y., June 21, 1932, Dean G. W. Mullins.

The dates of the summer meeting in Chicago in 1933 were tentatively set as June 19–24.

The titles and cross references to abstracts of the papers read at the sessions follow below. The papers whose abstract numbers are followed by *t* were read by title, the rest in person. The papers were read as follows: papers numbered 1 to 5 before a general session on Tuesday afternoon at which Vice-President W. A. Manning presided; papers numbered 6–32 were read before a section in Analysis on Wednesday afternoon with Professor W. E. Milne presiding; papers numbered 33–53 were read before a section in Algebra and Number Theory on Wednesday afternoon at which Professor D. N. Lehmer presided; and papers numbered 54–78 before a general session on Friday morning at which Professor M. I. Logsdon and Professor Dunham Jackson presided. Dr. Schoenberg presented paper No. 8, Dr. Martin presented paper No. 11, Professor Whyburn presented paper No. 13, Professor Ingraham presented paper No. 37, and Mr. Botsford presented paper No. 59. Dr. Martin was introduced by Professor Michal, Mr. Deutsch by Professor G. A. Pfeiffer, Professor Cauer by Professor J. D. Tamarkin, Professor J. H. Binney by Professor G. C. Evans, Mr. Hull by Professor L. E. Dickson, Mr. Clifford and Mr. Botsford by Professor Michal, and Dr. Sperner by Professor Wilhelm Blaschke.

1. *Diophantine equations from algebraic invariants and covariants*, by Professor E. T. Bell. (Abstract No. 38–7–186.)

2. *The degree and class of multiply transitive groups*, III, by Professor W. A. Manning. (Abstract No. 38–9–200.)

3. *Orthogonal trigonometric sums*, by Professor Dunham Jackson. (Abstract No. 38–7–191.)

4. *Note on a homogeneity property*, by Professor G. T. Whyburn. (Abstract No. 38–9–217.)

5. *On Nicod's reduction in the number of primitives of logic*, by Professor B. A. Bernstein. (Abstract No. 38–11–238.)

6. *A proof of E. H. Moore's equivalence*, by Professor J. P. Ballantine. (Abstract No. 38–7–174.)

7. *On the approximate solution of linear differential equations with boundary conditions*, by Professor W. H. McEwen. (Abstract No. 38–7–180.)

8. *On linear functional operations and the moment problem for a finite interval in one or several dimensions*, by Professor T. H. Hildebrandt and Dr. I. J. Schoenberg. (Abstract No. 38–9–198.)

9. *A classification of the critical sets for functions*, by Professor W. M. Whyburn. (Abstract No. 38–7–185.)

10. *Invariant functionals of functional forms*, by Professor A. D. Michal and Dr. R. S. Martin. (Abstract No. 38–9–219–*t*.)

11. *A set of expansions in vector space*, by Professor A. D. Michal and Dr. R. S. Martin. (Abstract No. 38–11–235.)

12. *An extension of a certain theorem of Kronecker*, by Mr. H. M. Bacon. (Abstract No. 38–9–201.)

13. *An application of the Dedekind cut notion to integration*, by Professors E. R. Hedrick and W. M. Whyburn. (Abstract No. 38–7–181.)

14. *On finite-rowed systems of linear inequalities in infinitely many variables*, II, by Dr. I. J. Schoenberg. (Abstract No. 38–9–216.)

15. *Particular solutions of* $y''+c_1x^py'+c_2x^{p-1}y=0$, *which consist of the product of a polynomial by an exponential*, by Professor James McGiffert. (Abstract No. 38–9–224.)

16. *On analytic complexes*, by Professor Solomon Lefschetz and Dr. J. H. C. Whitehead. (Abstract No. 38–7–189–*t*.)

17. *Note on a theorem due to Bromwich*, by Dr. H. L. Garabedian. (Abstract No. 38–5–158–*t*.)

18. *Functions satisfying a generalized Lipschitz condition*, by Mr. J. G. Deutsch. (Abstract No. 38–7–162–*t*.)

19. *Systems of linear difference equations and expansions in series of exponential functions*, by Professor R. D. Carmichael. (Abstract No. 38–7–171–*t*.)

20. *Systems of three linear partial differential equations of the third order*, by Mr. G. P. Wadsworth. (Abstract No. 38–7–184–*t*.)

21. *Differential fields and ideals of differential forms*, by Mr. H. W. Raudenbush, Jr. (Abstract No. 38–7–187–*t*.)

22. *Applications of an abstract existence theorem to both differential and difference equations*, by Dr. O. K. Bower. (Abstract No. 38–7–159–*t*.)

23. *A system of simultaneous bilinear functional equations*, by Dr. O. K. Bower and Mr. J. D. Grant. (Abstract No. 38–7–160–*t*.)

24. *Summation of functions of a complex variable*, by Professor R. D. Carmichael. (Abstract No. 38–5–156–*t*.)

25. *On the summability of double Fourier series*, by Professor H. L. Miller. (Abstract No. 38–9–195–*t*.)

26. *On the summability of triple Fourier series*, by Miss Alta Odoms. (Abstract No. 38–9–209–*t*.)

27. *On extremal surfaces*, by Dr. Max Coral. (Abstract No. 38–9–210–*t*.)

28. *On continued fractions in which the coefficients have limiting values*, by Professor H. S. Wall. (Abstract No. 38–9–218–*t*.)

29. *Remarks on continued fractions*, by Professor Raymond Garver. (Abstract No. 38–9–215–*t*.)

30. *The Poisson integral for functions with positive real part*, by Dr. W. Cauer. (Abstract No. 38–9–222–*t*.)

31. *An elliptic system of integral equations on summable functions*, by Professor J. H. Binney. (Abstract No. 38–11–237.)

32. *Note on the gradient of the Green's function*, by Professor G. C. Evans. (Abstract No. 38–11–236.)

33. *A number theoretic machine*, by Dr. D. H. Lehmer. (Abstract No. 38–9–202.)

34. *On representation of integers by ternary forms*, by Dr. A. E. Ross. (Abstract No. 38–9–208.)

35. *On Fermat's last theorem*, by Professor Glenn James. (Abstract No. 38–9–199.)

36. *The arithmetical theory of linear recurring series*, by Professor Morgan Ward. (Abstract No. 38–9–204.)

37. *Certain aspects of the theory of equations for a pair of matrices*, by Professor G. S. Bruton. (Abstract No. 38–9–196.)

38. *A study of certain related pairs of square matrices*, by Professor M. H. Ingraham. (Abstract No. 38–9–197.)

39. *Concerning primitive groups of class u*, by Dr. C. F. Luther. (Abstract No. 38–9–193.)

40. *Non-cyclic normal division algebras of degree and exponent four*, by Professor A. A. Albert. (Abstract No. 38–7–164–*t*.)

41. *A note on normal division algebras of order sixteen*, by Professor A. A. Albert. (Abstract No. 38–7–190–*t*.)

42. *A type of abstract arithmetic allied to a Boolean algebra*, by Professor Morgan Ward. (Abstract No. 38–9–206–*t*.)

43. *The cancellation law in the theory of congruences to a double*

modulus, by Professor Morgan Ward. (Abstract No. 38–9–205–*t*.)

44. *An inversive algorithm*, by Dr. D. H. Lehmer. (Abstract No. 38–9–203–*t*.)

45. *The numbers of solutions of congruences involving only kth powers*, by Mr. Ralph Hull. (Abstract No. 38–7–177–*t*.)

46. *On general Kronecker-(integer)-synthesis of disciplines*, by Dr. A. L. Foster. (Abstract No. 38–7–176–*t*.)

47. *Note on polynomials in a Galois field*, by Dr. Leonard Carlitz. (Abstract No. 38–7–168–*t*.)

48. *On the representation of a polynomial in a Galois field as the sum of an even number of squares*, by Dr. Leonard Carlitz. (Abstract No. 38–7–169–*t*.)

49. *On the representation of a polynomial as the sum of an odd number of squares*, by Dr. Leonard Carlitz. (Abstract No. 38–7–170–*t*.)

50. *On criteria for universality of ternary quadratic forms*, by Dr. A. E. Ross. (Abstract No. 38–9–192–*t*.)

51. *Groups* $\{S, T\}$ *whose commutator subgroups are abelian*, by Professor H. R. Brahana. (Abstract No. 38–7–166–*t*.)

52. *Prime-power abelian groups generated by a set of conjugates under a special automorphism*, by Professor H. R. Brahana. (Abstract No. 38–7–167–*t*.)

53. *On real quadratic fields*, by Dr. Charles Wexler. (Abstract No. 38–9–225–*t*.)

54. *On some properties of polygons related to the cuspidal cubic and a line*, by Professor Clifford Bell. (Abstract No. 38–5–153.)

55. *Geometry of a normed abelian group manifold*, by Mr. A. H. Clifford. (Abstract No. 38–9–221.)

56. *Involutorial space Cremona transformations determined by non-linear null reciprocities*, by Dr. E. J. Purcell. (Abstract No. 38–9–207.)

57. *The Apollonian spheres of a tetrahedron*, by Professor N. A. Court. (Abstract No. 38–7–183.)

58. *Simultaneous differential invariants of an affine connection and a general linear connection*, by Professor A. D. Michal and Mr. J. L. Botsford. (Abstract No. 38–9–220–*t*.)

59. *Normal representations and linear connections*, by Professor A. D. Michal and Mr. J. L. Botsford. (Abstract No. 38–11–234.)

60. *The place of Ananiah Shiragooni in the history of mathematics*, by Professor A. A. Shaw. (Abstract No. 38–9–212.)

61. *The Edgeworth taxation phenomenon*, by Professor Raymond Garver. (Abstract No. 38–9–214.)

62. *Misuse of postulations in theory of viscosity*, by Mr. B. F. Groat. (Abstract No. 38–5–155.)

63. *On topological transformations of a plane without invariant points*, by Dr. E. Sperner. (Abstract No. 38–11–232.)

64. *Possibilities for a universe with non-negative pressure*, by Professor H. P. Robertson. (Abstract No. 38–11–233.)

65. *Alternant surfaces*, by Professor Clifford Bell. (Abstract No. 38–5–152–*t*.)

66. *On 2-congruent graphs*, by Dr. Hassler Whitney. (Abstract No. 38–5–157–*t*.)

67. *On reduction of space Cremona involutions*, by Miss Esther McCormick. (Abstract No. 38–7–165–*t*.)

68. *Types of involutorial space transformations associated with certain rational curves*, by Mr. Amos Black. (Abstract No. 38–7–175–*t*.)

69. *On certain associated metric spaces*, by Mr. C. A. Lovell. (Abstract No. 38–7–163–*t*.)

70. *Continuous transformations of finite spaces*, by Dr. Rothwell Stephens. (Abstract No. 38–9–194–*t*.)

71. *Note on triple systems*, by Professor R. D. Carmichael. (Abstract No. 38–7–179–*t*.)

72. *New sets of independent postulates for the algebra of logic, with special reference to Whitehead and Russell's Principia Mathematica*, by Professor E. V. Huntington. (Abstract No. 38–7–182–*t*.)

73. *A set of postulates for the foundation of logic*, Second paper, by Professor Alonzo Church. (Abstract No. 38–7–161–*t*.)

74. *Remarks on propositions* *1.1 *and* *3.35 *of Principia Mathematica*, by Professor B. A. Bernstein. (Abstract No. 38–9–211–*t*.)

75. *Some properties of equality in combinatory logic*, by Dr. H. B. Curry. (Abstract No. 38–5–154–*t*.)

76. *Permanent configurations in the problem of four bodies*, by Professors W. D. MacMillan and Walter Bartky. (Abstract No. 38–7–178–*t*.)

77. *Concerning the limits of a measure of skewness*, by Professor Raymond Garver. (Abstract No. 38–9–213–*t*.)

78. *Concerning the mapping of locally connected continua on to simple arcs*, Preliminary communication, by Miss Beatrice Aitchison. (Abstract No. 38–11–231.)

M. H. INGRAHAM, *Associate Secretary*

INTERNATIONAL CONGRESS OF MATHEMATICIANS, ZURICH, 1932

BY R. G. D. RICHARDSON

Mathematicians from 37 countries to the number of approximately 650 together with 250 members of their families assembled in Zurich, Switzerland, for the Congress held September 4–12. As would be expected, the largest delegation of participants was from Switzerland (140), followed in order by Germany (111), France (68), United States and Canada (68), Italy (63), and Great Britain (38). The total attendance from this country and Canada, including members of the families of the active participants, was 104. Harvard and Brown had the largest delegations, there being four mathematicians from each. Among those who attended were the following 68 mathematicians:

C. R. Adams, R. B. Adams, J. W. Alexander, R. C. Archibald, I. A. Barnett, Garrett Birkhoff, H. F. Blichfeldt, D. G. Bourgin, R. W. Brink, W. D. Cairns, L. D. Cummings, L. L. Dines, H. L. Dorwart, H. T. Engstrom, F. J. Feinler, A. L. Foster, O. E. Glenn, M. C. Graustein, W. C. Graustein, Margaret Gurney, Einar Hille, T. R. Hollcroft, A. E. Holman, Charles Hopkins, W. A. Hurwitz, B. O. Koopman, A. M. Lehr, Florence Lewis, R. G. Lubben, E. M. Luse, Wilhelm Maier, T. E. Mason, G. M. Merriman, U. G. Mitchell, C. N. Moore, T. W. Moore, D. S. Morse, Marston Morse, Oystein Ore, R. A. C. Paley, G. Y. Rainich, R. G. D. Richardson, W. H. Roever, Stanislaw Saks, Arthur Sard, L. L. Silverman, W. G. Simon, L. G. Simons, D. E. Smith, P. A. Smith, Virgil Snyder, M. E. Stark, M. H. Stone, E. B. Stouffer, J. L. Synge, J. D. Tamarkin, Stepan Timoshenko, A. W. Tucker, J. S. Turner, H. W. Tyler, Oswald Veblen, H. M. Walker, J. L. Walsh, D. W. Weeks, J. H. C. Whitehead, D. V. Widder, Norbert Wiener, F. B. Williams.

The morning hours were occupied by 21 one-hour addresses given by invitation. This list included two presented by Americans and these were among the very best in material and form.

Nearly 200 other papers, including 27 by Americans and Canadians, were presented at sectional sessions in the afternoons.

Eight sections were organized: I. Algebra and Number Theory, II. Analysis, III. Geometry, IV. Probability and Statistics, V. Astronomy and Technical Mathematics, VI. Mechanics and Mathematical Physics, VII. Philosophy and History, VIII. Pedagogy. In II there were three parallel programs going forward at the same time and in III and VI two each, making in all twelve groups meeting simultaneously.

It is of interest to note that the first International Mathematical Congress was held in Zurich in 1897 and that the president of that gathering, Dr. Geiser, was present at this meeting. The 1932 Congress was organized by the University and the Technical Institute of Zurich, and Professor Fueter of Zurich presided in admirable fashion. The first scientific session was presided over by Hilbert and he was given a great ovation.

The official representatives of the Society present at the Congress were Professors C. N. Moore, R. G. D. Richardson, E. B. Stouffer, and J. D. Tamarkin; those of the United States government were J. W. Alexander, H. F. Blichfeldt, Marston Morse, M. H. Stone, and Oswald Veblen; those of the Mathematical Association of America were R. C. Archibald, W. D. Cairns, and D. E. Smith.

The following is a list of the invited addresses.

I. R. Fueter, *Idealtheorie und Funktionentheorie.*

II. C. Carathéodory, *Über die analytischen Abbildungen durch Funktionen mehrerer Veränderlicher.*

III. G. Julia, *Essai sur le développement de la théorie des fonctions de variables complexes.*

IV. W. Pauli, *Mathematische Methoden der Quantenmechanik.*

V. N. Tschebotarëw, *Die Aufgaben der modernen Galois'schen Theorie.*

VI. T. Carleman, *Sur la théorie des équations intégrales linéaires et ses applications.*

VII. E. Cartan, *Sur les espaces riemanniens symétriques.*

VIII. L. Bieberbach, *Operationsbereiche von Funktionen.*

IX. M. Morse, *The calculus of variations in the large.*

X. E. Noether, *Hyperkomplexe Systeme in ihren Beziehungen zur kommutativen Algebra und zur Zahlentheorie.*

XI. H. Bohr, *Fastperiodische Funktionen einer komplexen Veränderlichen.*

XII. F. Severi, *La théorie générale des fonctions analytiques de plusieurs variables et la géométrie algébrique.*

XIII. R. Nevanlinna, *Über die Riemannsche Fläche einer analytischen Funktion.*

XIV. R. Wavre, *L'aspect analytique du problème des figures planétaires.*

XV. J. W. Alexander, *Some problems in topology.*

XVI. F. Riesz, *Sur l'existence de la dérivée des fonctions d'une variable réelle et des fonctions d'intervalle.*

XVII. G. Valiron, *Le théorème de Borel-Julia dans la théorie des fonctions méromorphes.*

XVIII. W. Sierpinski, *Sur les ensembles de points qu'on sait définir effectivement.*

XIX. S. Bernstein, *Sur les liaisons entre quantités aléatoires.*

XX. K. Menger, *Neuere Methoden und Probleme der Geometrie.*

XXI. J. Stenzel, *Anschauung und Denken in der klassischen Theorie der griechischen Mathematik.*

The list of short papers presented by Americans either in person or by title is as follows:

C. R. Adams and J. A. Clarkson, *On definitions of bounded variation for functions of two variables.*

Elizabeth B. Cowley, *Technical vocabularies for plane and solid geometry.*

Louise D. Cummings, *On a method of comparison for straight line nets.*

L. L. Dines, *On linear inequalities.*

A. L. Foster, *On general Kronecker-(integer)-synthesis of disciplines.*

Solomon Gandz, *On alphabetical numerals.*

O. E. Glenn, *The mechanics of the stability of a central orbit.*

Einar Hille and J. D. Tamarkin, (1) *The summation of Fourier series by Hausdorff means.* (2) *On the summability of Fourier series.*

T. R. Hollcroft, *The general web of surfaces and the space involution defined by it.*

Edward Kasner, (1) *Element transformations of space for which normal congruences of curves are invariant.* (2) *Conformality in*

connection with functions of two complex variables. (3) *Conformal geometry in the complex domain.* (4) *Curvature theorems in dynamics.*

Wilhelm Maier, *Über die Riemannsche Q-Funktion.*

E. C. Molina, *An expansion for Laplacian integrals in terms of incomplete gamma functions, and some applications.*

C. N. Moore, *On certain properties of the Fourier constants of L-integrable functions of two variables.*

Oystein Ore, *Theory of non-commutative polynomials.*

G. Y. Rainich, *Determination of matter and force components from the Riemann tensor.*

Stanislaw Saks, *On certain functionals.*

D. E. Smith, *Presidential address of the International Committee on the Teaching of Mathematics.*

J. J. Smith, *An expression for Green's function in generalized coordinates.*

Virgil Snyder, *On a series of Cremona involutions defined by a pencil of ruled surfaces.*

E. B. Stouffer, *On the projective differential geometry of developable surfaces.*

J. L. Synge, *The equilibrium of a tooth with a general conical root.*

J. H. C. Whitehead, *Locally homogeneous spaces in differential geometry.*

Norbert Wiener and R. A. C. Paley, *Analytic properties of characters of infinite abelian groups.*

Printed abstracts of the short papers on the preliminary program were available on the opening day. The Proceedings will be published and sent to those participating in the Congress; these will contain the invited addresses in full and abstracts of the short papers.

The International Committee on the Teaching of Mathematics under the presidency of Professor D. E. Smith held a session which was constituted as a meeting of Section VIII. This Committee which had been revived at the Bologna Congress elected Hadamard as president, he being the third in the line after Klein and Smith. Vice-presidents elected include Lietzmann, Scorza, and Heegaard; and Fehr was made secretary-treasurer with E. H. Levi as assistant. It is the purpose of the Committee in

the next quadrennium to study the trend of the teaching of mathematics.

At one of the business sessions a telegram of felicitations was received from the Society at its session in Los Angeles. A message was received from Picard stating his regret at not being able to be present and a cordial reply was sent. A message of congratulation was sent to Brill on the occasion of his 90th birthday.

Neville made a statement to the Congress regarding the fund available for publication of mathematical tables and suggested that any person contemplating the issuing of tables consult him.

A resolution of deep regret on the recent death of Professor J. C. Fields was adopted. Fields had proposed to establish two gold medals, to be awarded at successive Congresses for outstanding achievement in mathematics. This offer was accepted by the Congress, and the following committee was appointed to name the recipients of the medals to be awarded in 1936: Severi (chairman), Birkhoff, Carathéodory, Cartan, and Tagaki.

The term of the statutes of the International Mathematical Union having expired with 1931, representatives of the various countries had been invited to a session to discuss what steps, if any, should be taken to perpetuate the organization. It should be pointed out that this Congress was not held under the auspices of the Union. At a meeting of those Americans present at Zurich, Professors C. N. Moore, R. G. D. Richardson, Virgil Snyder, and Oswald Veblen were chosen to represent the delegation. The sentiment of the group crystallized around three points: (1) that the invitation to the next Congress should be accepted by the present Congress rather than by the Union; (2) that it would appear that a permanent international organization had no problems important enough to warrant its existence; (3) that if an international organization in mathematics were to continue, it should be divorced from connection with national governments. The meeting of the Union took place at 10:00 A.M. on September 11 and it was voted to request President Fueter to name to the Congress the personnel of an international commission to investigate the desirability of continuing an international mathematical organization and to report to the Congress at Oslo, the Union in the meantime to be in abeyance. The Congress appointed such a Commission, the personnel consist-

ing of Severi (chairman), Alexandroff, Bohr, Fejér, Julia, Mordell, Terradas, de la Vallée-Poussin, Veblen, Weyl, and Zaremba. To this Commission was referred also a query raised in the Congress regarding the desirability of an international committee on bibliography.

The beautiful city of Zurich was most hospitable and every arrangement was made for the comfort and entertainment of the visiting mathematicians. Excellent arrangements were made for caring for the visitors at moderate cost. Tuesday afternoon and Thursday were devoted to excursions to beautiful spots in the neighboring regions of Switzerland. On the evening of the 10th the State entertained the mathematicians and guests at a festival in the Stadttheater. President Fueter presided and among the speakers were Plancherel for the Zurich universities and Cartan, Severi, Veblen, and Weyl for the visitors. On the afternoon of the 11th the City entertained at a tea in a hotel in the suburbs. Several other entertainments were provided also, and a special program for the visiting ladies was carried out. A display of recent mathematical books and of mathematical instruments was a feature of the meeting.

Resolutions of thanks for the generous hospitality provided were carried unanimously on motion of Bortolotti for the Congress and of Madame Riabouchinsky for the ladies.

An invitation from Norway to hold the next Congress in Oslo was presented at the last session and was accepted unanimously.

BROWN UNIVERSITY

THOMAS HAKON GRONWALL—IN MEMORIAM

With Thomas Hakon Gronwall, who died in New York May 9, 1932, the Society has lost one of its most colorful personalities. By training, habit, and preference a mathematician to whom no chapter of mathematics was a closed book, he could hold his own with physical chemists, and had made a living as a civil engineer specializing in structural steel.

Hakon Tomi Grönwall was born January 16, 1877, at Dylta Bruk in the parish of Axberg in central Sweden. In 1893 he entered the University of Uppsala, but transferred to Stockholm the following year. He was attracted to Stockholms Högskola* by the vigorous and flourishing mathematical school which centered around Mittag-Leffler and which contained among its members mathematicians like Bendixson, Fredholm, von Koch, and Phragmén. That he thrived in this stimulating atmosphere is shown by the ten papers which he wrote in 1895–8†; their range is typical for the broadness of his mathematical interests. He received his doctor's degree at Uppsala at the age of 21; his thesis is [7] in the Bibliography at the end of this obituary. He left Stockholm and Sweden, probably in 1899, owing to a student's prank which was treated by the authorities, sadly lacking in sense of humor, in a way which hurt his youthful pride and his strong love of liberty.

Gronwall now started a completely new chapter. He studied engineering in Berlin, received his degree at the Charlottenburg Technische Hochschule in 1902, and began practising. From 1904 onwards he seems to have been located in this country. Very little is known about this period in Gronwall's life. He was apparently a rolling stone, stopping at the various steel works for longer or shorter periods. He was connected with the American Bridge Co., the Pennsylvania Railroad, and other engineering companies.

In 1911 he lived in Chicago and was evidently ready to make another right-about-face, but now back to his old love. Somehow he managed to keep up with the development in mathematics, so that when he made his spectacular come-back in 1912, it was as a full-fledged mathematician. A number of papers accepted for publication by leading periodicals during the spring of 1912 gave him a desirable confidence in his own abilities. Gronwall made his first appearance before the Society at the Chicago meeting on April 5 and 6, 1912, in which he took a prominent part; and he became a member of the Society during the same month. We can date his return to mathematics from these events.

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* High school is not a correct translation of the Swedish word *Högskola*. In the nineties Stockholms Högskola was essentially a research institute in mathematics and the natural sciences though lacking most of the equipment now-a-days considered necessary for such a task. The student body was small but select. No examinations were held nor degrees given until 1909; a student who desired such baubles had to run the gauntlet at Uppsala. Students who could work under such handicaps had to be earnest seekers of truth.

† These papers are signed H. Grönwall, all later papers have the signature T. H. Gronwall.

During the twenty years of his membership he made nearly ninety communications to the Society. His activities are not evenly distributed in time. The volcanic eruption which started in 1911–2 came to an end in 1916; it flared up again in 1919–20, but faded out after a few years. The fourth period of intense activity started in 1925 and lasted with undiminished strength until his final illness and death. These various periods reflect the vicissitudes in his external circumstances.

Gronwall was called to Princeton in 1913 as an instructor and was adjoined to the editorial board of the Annals. He was made an assistant professor in 1914, but withdrew in 1915. He gave excellent graduate courses, inspired research, and was most useful to the Annals as a contributor and as a critic. Though Gronwall left Princeton long ago, he never lost contact with the Princeton group; he remained on the board of the Annals until 1928 and was a faithful contributor until his premature death. The Annals got more than one third of all his papers, and during the last years of his life nearly all his papers in pure mathematics.

At the next turn of the wheel, we find Gronwall as a mathematical expert on the Technical Staff of the Chief of Ordinance, sharing his time between Washington and the Aberdeen Proving Grounds. This occupation, which lasted from 1918 until 1922, has left some traces in his publications [see 57, 61 and *9*, *11*]. After the demobilization, he established himself as a consulting mathematician in New York. He devoted part of his time to the interests of the American Telephone and Telegraph Co. [see 64], and used to complain that their problems required his knowing Watson's *Theory of Bessel's Functions* by heart. In 1925 he started to collaborate with Professor Victor K. La Mer, which led to his joining the Department of Physics at Columbia University as an associate in 1927. This connection seems to have suited him fully. There were no teaching obligations; he had complete control of his own time and an abundance of new intriguing problems to solve in physical chemistry and in atomic physics. During the last years of his life he worked steadily on the wave mechanics of the helium atom, but was apparently stricken down before he could achieve results of physical significance.*

This sketch of Gronwall's life is necessarily somewhat fragmentary. I shall now proceed to a summary account of his scientific production which I group according to subjects. Gronwall published 85 papers. In addition he communicated the results of 24 investigations to the Society, the details of which are still unpublished. These papers are listed in the Bibliography at the end, reference to which will be made by numbers in square brackets, italic numerals for unpublished, ordinary numerals for published papers. Gronwall was much in demand as a reviewer, having published 33 reviews in this Bulletin.

1. *Algebra and number theory.* [4, 5, 6, 72, 77, and *16*.] This field is not so well represented. His early investigations on transcendental numbers do not contain any new results, the other papers seem to be of minor interest.

2. *Analytical number theory.* [17, 19, 20, 21.] Here we are dealing with

* An inventory of his posthumous papers may prove this statement to be unduly pessimistic.

much more important papers, contributions which helped to make Gronwall famous in 1912–13. In [17] he is concerned with the asymptotic properties of the number theoretical function $\sigma_\alpha(n)$, that is, the sum of the αth powers of the divisors of n. In [19] and [20] he studies the Riemann zeta function, its reciprocal and its logarithmic derivative on the line $\sigma=1$. In [21] he applies the same methods in studying the order of magnitude of $L(1, \chi)$ as a function of k, where $L(s, \chi)$ is an L-function of Dirichlet corresponding to complex characters modulo k. Some of Gronwall's estimates have been improved upon later, others still represent the best available.

3. *Functions of a real variable.* [12, 13, 14, 22, 23, 24, 25, 26, 27, 28, 29, 40, 60, 73, 79, 83, and *4*, *14*.] With one or two exceptions all these investigations deal with Fourier series, Laplace series, and the theory of summability, in general or as applied to these series. Gronwall devoted several papers to Gibbs' phenomenon and its generalizations. Paper [12] contains a complete discussion of the phenomenon for the partial sums; [14] is a study of the behavior of the partial sums in the case of bounded functions whose Fourier coefficients are $O(1/n)$, where he encountered a similar phenomenon; [73] deals with the (C, k)-sums and contains the determination of Cramér's constant. Paper [79] deals with another peculiarity of the (C, k)-sums of Fourier or Laplace series. Lebesgue's constants, ρ_n, form the subject matter of [13] and [25]. Gronwall proved Fejér's guess that the sequence $\{\rho_n\}$ is monotone. His own guess that the sequence might be completely monotone was proved later by Szegö. Paper [60] contains an interesting complement to Carleman's theorem on the Fourier coefficients of continuous functions. The investigations on the series of Laplace and Legendre [22, 24, 26, and 28] belong to Gronwall's very best papers. Here he determined the order of magnitude of the corresponding Lebesgue constants, constructed functions exhibiting the du Bois-Reymond or the Lebesgue singularity, determined the index of Cesàro summability with careful discussion of the conditions at the antipode or at the end points, and studied the degree of convergence. Most results are final and represent a considerable advance beyond the previous results due to Chapman, Fejér, Haar, and Jackson. Gronwall showed that any series summable (C, k) is also summable de la Vallée Poussin [27] (simultaneously proved by C. N. Moore); his further studies of the (VP)-means [40 and *4*] ultimately led him to an elegant investigation of a class of definitions of summation defined by conformal mapping [83].

4. *General theory of functions of one complex variable.* [9, 15, 16, 31, 42, 43, 54, 59, 63, and *8*, *17*.] The range of these papers is impressive; their strength lies perhaps less in the novelty of the result than in the elegance of the proofs, however. As an example we may mention [31] and [63], which give new and elementary proofs of important theorems on power series with prescribed initial coefficients.

5. *Special analytic functions.* [36, 41, 44, 47, 66, 84, and *6*, *15*.] Gronwall translated Jensen's excellent paper on the gamma function [36]; to this he added his own exposition based on definite integrals [47], and several investigations of special questions in the theory of the gamma and related functions of which [66] is perhaps the most interesting.

6. *Conformal mapping.* [30, 37, 38, 51, 52, 55, 56, 58, and 7.] The main existence questions having been solved around 1912–13, a study of the fine structure of the mapping function could start. Gronwall took a prominent part in this study. Of particular importance is [30] which involves applications of the area theorem, partly contemporary with, partly preceding the corresponding investigations of Bieberbach. The other notes contain a number of interesting estimates; his claim in [7] of having solved the coefficient problem was probably based upon an error, however. It is a pity that Gronwall did not publish a comprehensive presentation of his work in this field.*

7. *Functions of several complex variables.* [1, 2, 3, 7, 45, 62, and *1, 3, 11.*] Gronwall's liking for this field goes back to 1895. Paper [1] contains among other things a generalization of Laurent's theorem, restated in [7] and [3]; [7] also contains an important contribution to the notion of irreducibility. The theory of abelian functions and integrals plays a great rôle in his early work; [2] gives a mode of constructing integrals of the first kind. But Gronwall's most important contribution to this field is [45] in which he takes up Cousin's work on the expressibility of meromorphic functions as the quotient of entire functions and the extensions of the theorems of Weierstrass and Mittag-Leffler for critical examination; he restates and corrects Cousin's main theorems, and proves by an example that the original formulations were at fault.

8. *Differential equations.* [1, 3, 7, 8, 10, 49, 50, 70, 75, 85, and *2, 20.*] Here [50] and [70] deal with ordinary differential equations, [49, 75, 85, and *20*] with partial ones. The main interest attaches to his early investigations on systems of m linear total differential equations in m dependent and n independent variables whose general solution can be expressed linearly in terms of a fundamental system of m^2 functions. The singularities of the single-valued coefficients are supposed to be the manifolds on which certain entire functions vanish. The Fuchs-Frobenius theory can be extended to this class of equations. Gronwall gives a reduction to normal form, studies the solutions in the neighborhood of a point on one of the singular manifolds, and determines conditions for the manifold to be regular-singular. These investigations complete and extend earlier, partly unsatisfactory, research in the case $n=2$ and rational coefficients due to Fuchs, Horn, and others. In [10] he considers the case in which a point lies on several singular manifolds and obtains a sufficient condition that the corresponding substitutions commute, and proves by an example that Horn's discussion of a special case of this question is erroneous. In several papers he treats the case in which the coefficients are $2n$-tuply periodic and the general solution is single-valued. Gronwall planned at least twice to republish these investigations in more accessible form, but unfortunately never did so. As a consequence they seem to have been overlooked by later writers.

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* Perhaps he wrote such a paper; I have at least a vague recollection of H. von Koch telling me in 1921 that Gronwall had submitted such a memoir to the Acta Mathematica in competition for the prize later awarded to Koebe. The merits or demerits of this memoir and its ultimate fate are matters beyond my ken.

9. *Differential geometry.* [34, 35, 80, 85, and *5, 21.*] The differential geometric questions treated by Gronwall are scarcely of fundamental importance, but they led to difficult analytic problems which required all his skill. For this reason, if for no other, these papers demand our respect.

10. *Mathematical physics* (see also sections 11 and 12). [18, 33, 39, 46, 53, 57, 61, 65, 69, 76, 81, and *9, 10, 11.*] The papers on elasticity [46, 53, 69] reflect the interest he held for such questions since his engineering days. The papers on ballistics [57, 61, and *9, 11*] date from the war period, the paper on induction [64] is an A. T. & T. Co. product. His work in potential theory [18, 81, and *10*] is of considerable interest. Two notes on the kinetic theory of gases [33, 39] and two on optics [65, 76] add to the kaleidoscopic effect.

11. *Atomic physics.* [77, 78, 85, and *22, 23.*] Gronwall's important work in this field was interrupted by his premature death. Papers [77, 78] deal with the wave mechanics of the hydrogen atom; [85] is a mathematical complement to a memoir on the helium atom intended for the Physical Review. A paper elaborating the results in the lattice theory of crystals announced in [*22*] is expected to appear later.

12. *Physical chemistry.* [67, 68, 70, 71, 82 and *18, 24.*] For these questions I am happy to be able to quote from a letter by Professor Victor K. La Mer who writes:

"Physical chemists are deeply indebted to Dr. Gronwall for the interest and rare mathematical skill which he brought to bear in solving the problem of the anomaly of strong electrolytes. This problem has been the outstanding stumbling block of physical chemistry since the original formulation of the dissociation theory by Arrhenius in 1888. In 1923 Debye and Hückel developed a theory which completely revolutionized the point of view. Their theory amounts to determining the correction to the equation of state for a system of electrically charged particles in a medium of dielectric constant D by determining the excess electric potential arising from the unequal distribution of the ions. The theory finds mathematical expression in terms of a combination of the Poisson and Boltzmann equations. Debye and Hückel, however, neglected all terms of higher order than the linear term in the expansion of the exponential in their solution of the problem. By 1925 sufficient experimental evidence had accumulated to convince the writer that the theory would have to be abandoned despite its many remarkable successes unless it could be shown that the very marked discrepancies which frequently persisted to extreme dilutions in the most significant data could be attributed to an incomplete mathematical development. Gronwall immediately grasped the physical significance of the problem; the mathematical aspects were sufficiently difficult to attract his intense interest. Paper [71] contains a complete analytical solution of the Poisson-Boltzmann equation, along with extensive tables for evaluating the numerical solution in the third and fifth approximation valid for the case of salt mixtures of symmetrical valence types. Paper [70] contains an existence proof of the solution and in [82] the complicated numerical methods for the unsymmetric case are completed and confirmed. Gronwall's solution has since been tested extensively on experimental work from this

laboratory and elsewhere. It reproduces the results with remarkable fidelity and completely removes the manifest absurdity of negative ion sizes inherent in the original theory. Only a mathematician with Gronwall's gift for analysis and most uncommon grasp of the literature of chemistry and physics could have contributed the elegant solution which he gave. His achievement will remain as a worthy monument of the services which mathematics can render chemistry and physics."

"In our several years of close association, Gronwall was never too busy to instruct the writer or his collaborators in elementary and tedious details of mathematical technique. The skill with which he could analyze and solve difficult problems excited the wonder and admiration of his colleagues. The clarity of his mind and writings have been an inspiration to every one associated with him."

13. *Miscellaneous.* [11, 32, 48, 74, and *13.*] Paper [11] is a fundamental memoir on nomography. The others are minor notes on integral equations, non-euclidean geometry, convex solids, and statistics.

After this survey let us try to analyze Gronwall as a scientific writer. The first impression is undoubtedly that of bewildering versatility. A closer examination reveals, however, that there is much more unity in this work than is apparent at first sight. Gronwall was first and last an analyst, but he frequently went to other fields for questions to turn into analytical problems. His command of the classical tools of analysis was superb; he worked in practically all the main fields of analysis and left a mark for himself in several. His knowledge of the literature, even in remote fields, was remarkable. He was a highly critical reader who got more than one interesting paper out of flaws detected in the current literature. It is idle to speculate over what he might have done, had he done less or lived longer. His life shows that his unruly spirit found expression, joy, and satisfaction in scientific thinking and creation. The average degree of interest of his papers is very high, even those of less intrinsic importance are never empty, and his creations all carry the earmark of an artist who prided himself on doing his work thoroughly, accurately, and with elegance. His papers are usually very well written, he never sacrifices clarity for brevity, and he plays fair with the reader. Gronwall liked to get an elegant solution, but he was able to master great formal complications if necessary. His long and frequent contacts with the applications had given him a strong feeling for what constituted a useful solution in such fields; he was himself an experienced and skilled computer.

When I first met Gronwall some eleven years ago, he was already a disillusioned man, modest, quiet, and retiring. Though he must have been occasionally somewhat of a trial to the puritanical brethren, I have never heard of his having any enemies. His friends found him fair-minded, kind-hearted, and unselfish. He was always willing to put his great knowledge and skill at the service of colleagues, friends, and students, and was eagerly interested in their problems. His non-scientific interests were chiefly intellectual, his acquaintance with general literature was both thorough and wide. His death at the height of his powers means a great loss to science and to his many friends.

Bibliography

Of the subsequent lists, the first contains the published papers, the second one the abstracts from this Bulletin of the unpublished ones. The abbreviations used in the first list are explained at the end of this paper. The notation [19 (9-11-12) 61] means that this paper was presented to the Society September 11, 1912, an abstract having appeared on page 61 of volume 19 of this Bulletin.

I. Published Papers

1. *Om system af lineära totala differentialekvationer*, ÖKVAF, vol. 52 (1895), pp. 729–757.
2. *Ueber Integrale algebraischer Differentialausdrücke von mehreren Veränderlichen*, ÖKVAF, vol. 53 (1896), pp. 67–72.
3. *Några användnigar af de 2n-periodiska funktionerna på teorin för system af lineära totala differentialekvationer*, ÖKVAF, vol. 53 (1896), pp. 295–314.
4. *Note sur les fonctions et les nombres algébriques*, ÖKVAF, vol. 54 (1897), pp. 199–203.
5. *Deux théorèmes sur les nombres transcendants*, ÖKVAF, vol. 54 (1897), pp. 623–632.
6. *Sur les nombres transcendants*. II. ÖKVAF, vol. 55 (1898), pp. 153–156.
7. *Om system af linjära totala differentialekvationer särskilt sådana med 2n-periodiska koefficienter*. Akademisk afhandling. Uppsala, 1898, 52 pp.
8. *Eine Verallgemeinerung der Lamé'schen Differentialgleichung*, ÖKVAF, vol. 55 (1898), pp. 357–368.
9. *Sur les fonctions qui ne satisfont à aucune équation différentielle algébrique*, ÖKVAF, vol. 55 (1898), pp. 387–395.
10. *Sur les singularités des systèmes d'équations linéaires aux différentielles totales*, BKSVAH, vol. 24 (I:9) (1899), 22 pp.
11. *Sur les équations entre trois variables représentables par des nomogrammes à points alignés*, JdM, (6), vol. 8 (1912), pp. 59–102.
12. *Über die Gibbssche Erscheinung und die trigonometrischen Summen* $\sin x + \frac{1}{2}\sin 2x + \cdots + (1/n)\sin nx$, MA, vol. 72 (1912), pp. 228–243.
13. *Über die Lebesgueschen Konstanten bei den Fourierschen Reihen*, MA, vol. 72 (1912), pp. 244–261.
14. *On a theorem of Fejér's and an analogon to Gibbs' phenomenon*, TAMS, vol. 13 (1912), pp. 445–468. [18 (4-5-12) 445-6.]
15. *Sur un théorème de M. Picard*, CR, vol. 155 (1912), pp. 764–6. [19 (11-30-12) 225–6.]
16. *On analytic functions of constant modulus on a given contour*, AM, (2), vol. 14 (1912), pp. 72–80. [19 (9-11-12) 61.]
17. *Some asymptotic expressions in the theory of numbers*, TAMS, vol. 14 (1913), pp. 113–122. [19 (4-6-12) 447.]
18. *Some special boundary problems in the theory of harmonic functions*, BAMS, (2), vol. 19 (1913), pp. 227–233. [19 (9-11-12) 61.]

19. *Über das Verhalten der Riemannschen Zetafunktion auf der Geraden* $\sigma = 1$, AMP, (3), vol. 20 (1913), pp. 231–8. [19 (11–30–12) 224.]
20. *Sur la fonction* $\zeta(s)$ *de Riemann au voisinage de* $\sigma = 1$, RCMP, vol. 35 (1913), pp. 95–102. [19 (11-30-12) 225.]
21. *Sur les séries de Dirichlet correspondant à des caractères complexes*, RCMP, vol. 35 (1913), pp. 145–159. [19 (11-30-12) 225.]
22. *Über die Laplacesche Reihe*, MA, vol. 74 (1913), pp. 213–270. [19 (9-10-12) 61–2.]
23. *On the summability of Fourier's series*, BAMS, (2), vol. 20 (1913), pp. 139–146. [19 (2-22-13) 392.]
24. *On the degree of convergence of Laplace's series*, TAMS, vol. 15 (1914), pp. 1–30. [19 (2-22-13) 392.]
25. *On Lebesgue's constants in the theory of Fourier series*, AM, (2), vol. 15 (1914), pp. 125–8. [19 (2-22-13) 392.]
26. *Sur la série de Laplace*, CR, vol. 158 (1914), pp. 1488–90. [19 (1-2-13) 299.]
27. *Sur quelques méthodes de sommation et leur application à la série de Fourier*, CR, vol. 158 (1914), pp. 1664–5. [19 (3-21-13) 442.]
28. *Über die Summierbarkeit der Reihen von Laplace und Legendre*, MA, vol. 75 (1914), pp. 321–375. [19 (1-2-13) 299.]
29. *On approximation by trigonometric sums*, BAMS, (2), vol. 21 (1914), pp. 9–14. [20 (12-31-13) 299.]
30. *Some remarks on conformal representation*, AM, (2), vol. 16 (1914), pp. 72–76, and p. 138. [21 (10-31-14) 165.]
31. *On the maximum modulus of an analytic function*, AM, (2), vol. 16 (1914), pp. 77–81. [19 (4-26-13) 507.]
32. *An integral equation of the Volterra type*, AM, (2), vol. 16 (1914), pp. 119–122. [21 (1-2-15) 284.]
33. *A functional equation in the kinetic theory of gases*, AM, (2), vol. 17 (1916), pp. 1–4. [21 (2-24-15) 484.]
34. *Sur les surfaces minima formant une famille de Lamé*, CR, vol. 161 (1915), pp. 582–3. [20 (4-25-14) 517.]
35. *Determination of all triply orthogonal systems containing a family of minimal surfaces*, AM, (2), vol. 17 (1916), pp. 76–100. [20 (4-25-14) 517.]
36. *An elementary exposition of the theory of the gamma function*. Annotated translation of a Danish memoir by J. L. W. V. Jensen. AM, (2), vol. 17 (1916), pp. 124–166.
37. *Sur la déformation dans la représentation conforme*, CR, vol. 162 (1916), pp. 249–252. [21 (1-2-15) 285; 22 (2-26-16) 376.]
38. *Sur la déformation dans la représentation conforme sous des conditions restrictives*, CR, vol. 162 (1916), pp. 316–318. [22 (2-26-16) 376.]
39. *Sur une équation fonctionnelle dans la théorie cinétique des gaz*, CR, vol. 162 (1916), pp. 415–418. [22 (2-26-16) 375.]

40. *Über einige Summationsmethoden und ihre Anwendung auf die Fouriersche Reihe*, JfM, vol. 147 (1916), pp. 16–35. [19 (3-21-13) 442.]

41. *Sur les zéros des fonctions $P(z)$ et $Q(z)$ associées à la fonction gamma*, ASENS, (3), vol. 33 (1916), pp. 381–393. [22 (2-26-16) 375.]

42. *A problem in geometry connected with the analytic continuation of a power series*, AM, (2), vol. 18 (1916), pp. 65–69. [23 (9-5-16) 81.]

43. *On the power series for* $\log(1+z)$, AM, (2), vol. 18 (1916), pp. 70–73. [23 (9-5-16) 80.]

44. *On the convergence of Binet's factorial series for* $\log \Gamma(s)$ *and* $\psi(s)$, AM, (2), vol. 18 (1916), pp. 74–78. [23 (9-5-16) 81.]

45. *On the expressibility of a uniform function of several variables as the quotient of two functions of entire character*, TAMS, vol. 18 (1917), pp. 50–64. [20 (10-25-13) 173–174.]

46. *Elastic stresses in an infinite solid with a spherical cavity*, AM, (2), vol. 19 (1918), pp. 295–296. [22 (2-26-16) 379.]

47. *The gamma function in the integral calculus*, AM, (2), vol. 20 (1918), pp. 35–124.

48. *On the shortest line between two points in non-euclidean geometry*, AM, (2), vol. 20 (1918), pp. 200–201.

49. *On a system of linear partial differential equations of the hyperbolic type*, AM, (2), vol. 20 (1918), pp. 274–278.

50. *Note on the derivatives with respect to a parameter of the solutions of a system of differential equations*, AM, (2), vol. 20 (1918), pp. 292–296.

51. *A theorem on power series, with an application to conformal mapping*, PNAS, vol. 5 (1919), pp. 22–24. [25 (12-28-18) 256.]

52. *On the twist in conformal mapping*, PNAS, vol. 5 (1919), pp. 248–250.

53. *On the influence of keyways on the stress distribution in cylindrical shafts*, TAMS, vol. 20 (1919), pp. 234–244. [22 (2-26-16) 379.]

54. *Investigation of a class of fundamental inequalities in the theory of analytic functions*. Annotated translation of a Danish paper by J. L. W. V. Jensen. AM, (2), vol. 21 (1919), pp. 1–29.

55. *On the distortion in conformal mapping when the second coefficient in the mapping function has an assigned value*, PNAS, vol. 6 (1920), pp. 300–302. [26 (2-28-20) 340.]

56. *Conformal mapping of a family of real conics on another*, PNAS, vol. 6 (1920), pp. 312–315.

57. *Qualitative properties of the ballistic trajectory*, AM, (2), vol. 22 (1920), pp. 44–65. [25 (4-26-19) 441; 26 (2-28-20) 340.]

58. *Conformal mapping of a family of real conics upon another*, AM, (2), vol. 22 (1920), pp. 101–127.

59. *A sequence of polynomials connected with the nth roots of unity*, BAMS, (2), vol. 27 (1921), pp. 275–279. [27 (9-8-20) 64–65.]

60. *On the Fourier coefficients of a continuous function*, BAMS, (2), vol. 27 (1921), pp. 320–321. [27 (9-8-20) 64.]

61. *Differential variations in ballistics, with applications to the qualitative properties of the trajectory*, TAMS, vol. 22 (1921), pp. 505–525. [26 (10-25-19) 149.]

62. *Summation of a double series*, AM, (2), vol. 23 (1922), pp. 282–285. [27 (2-26-21) 349.]

63. *On power series with positive real part in the unit circle*, AM, (2), vol. 23 (1922), pp. 317–332. [28 (9-8-21) 9–10.]

64. *The mutual inductance of two square coils*, TAMS, vol. 27 (1925), pp. 516–536. [29 (4-28-23) 208.]

65. *Reflection of radiation from a finite number of equally spaced parallel planes*, PR, (2), vol. 27 (1926), pp. 277–285. [28 (2-25-22) 239.]

66. *On the zeros of the function $\beta(z)$ associated with the gamma function*, TAMS, vol. 28 (1926), pp. 391–399, and p. 786. [23 (9-5-16) 81.]

67. *On the extension of the Debye-Hückel theory of strong electrolytes to concentrated solutions.* Written with V. K. La Mer. S, vol. 64 (1926), p. 122. [32 (5-1-26) 316.]

68. *On the determination of the apparent diameters of the ions in the Debye-Hückel theory of strong electrolytes*, PNAS, vol. 13 (1927), pp. 198–202.

69. *The longitudinal vibrations of a liquid contained in a tube with elastic walls*, PR, (2), vol. 30 (1927), pp. 71–83. [33 (2-26-27) 258.]

70. *On the existence and properties of the solutions of a certain differential equation of the second order*, AM, (2), vol. 28 (1927), pp. 355–364. [32 (1-2-26) 127; 33 (2-26-27) 258.]

71. *Über den Einfluss der sogenannten höheren Glieder in der Debye-Hückelschen Theorie der Lösungen starker Elektrolyte.* Written with Victor K. La Mer and Karl Sandved. PZ, vol. 29 (1928), pp. 358–393. [32 (10-30-26) 316.]

72. *The number of arithmetical operations involved in the solution of a system of linear equations*, AMM, vol. 36 (1929), pp. 325–327. [35 (3-30-29) 443.]

73. *Zur Gibbsschen Erscheinung*, AM, (2), vol. 31 (1930), pp. 233–240. [31 (2-28-25) 302; 32 (5-1-26) 316.]

74. *On Minkowski's mixed volume of three convex solids*, AM, (2), vol. 31 (1930), pp. 470–2. [27 (6-17-20) 1, title but no abstract.]

75. *The converse of Euler's theorem on homogeneous functions*, AM, (2), vol. 31 (1930), pp. 743–744. [33 (2-26-27) 258–259.]

76. *A formula in geometrical optics*, AM, (2), vol. 31 (1930), pp. 475–478. [36 (10-26-29) 51.]

77. *A diophantine equation connected with the hydrogen spectrum*, PR, (2), vol. 36 (1930), pp. 1671–1672. [33 (10-30-26) 12.]

78. *On the wave equation of the hydrogen atom*, AM, (2), vol. 32 (1931), pp. 47–52. [34 (9-6-28) 703.]

79. *On the Cesàro sums of Fourier's and Laplace's series*, AM, (2), vol. 32 (1931), pp. 53–59. [31 (10-25-24) 109–110.]

80. *A functional equation in differential geometry*, AM, (2), vol. 32 (1931), pp. 313–326. [25 (12-28-18) 255.]

81. *Über den Konvergenzbereich der Potenzreihenentwicklung einer harmonischens Funktion von n Veränderlichen*, MZ, vol. 33 (1931), pp. 177–185. [32 (1-2-26) 127.]

82. *The influence of the higher terms of the Debye-Hückel theory in the case of unsymmetric valence type electrolytes.* Written with V. K. La Mer and Lotti J. Greiff. JPC, vol. 35 (1931), pp. 2245–2288, and pp. 3103–3104.

83. *Summation of series and conformal mapping*, AM, (2), vol. 33 (1932), pp. 101–117. [32 (10-31-25) 36.]

84. *An inequality for the Bessel functions of the first kind with imaginary arguments*, AM, (2), vol. 33 (1932), pp. 275–278. [37 (9-8-31) 530.]

85. *A special conformally euclidean space of three dimensions occurring in wave mechanics*, AM, (2), vol. 33 (1932), pp. 279–293. [37 (9-8-31) 530.]

II. Abstracts of Unpublished Papers

1. *On Weierstrass's preparation theorem.* [19 (1-2-13) 298.]
2. *On systems of linear total differential equations.* [20 (12-31-13) 299.]
3. *Extension of Laurent's theorem to several variables.* [20 (12-31-13) 299.]
4. *On the summation method of de la Vallée Poussin.* [21 (1-2-15) 284.]
5. *Equipotential minimal surfaces.* [25 (12-28-18) 256.]
6. *On Kummer's series.* [25 (12-28-18) 256.]
7. *Upper bounds for the coefficients in conformal mapping.* [27 (9-8-20) 65.]
8. *Some inequalities in the theory of functions of a complex variable.* [27 (10-30-20) 103.]
9. *Some empirical formulas in ballistics.* [27 (2-26-21) 349.]
10. *On biharmonic functions.* [28 (10-29-21) 93.]
11. *Qualitative properties of the ballistic trajectory* (second paper). [28 (2-25-22) 239.]
12. *Isothermal surfaces with spherical lines of curvature in one system.* [29 (4-28-23) 218.]
13. *Extension of Tchebychef's statistical theorem.* [29 (4-28-23) 218.]
14. *Extension of a theorem due to Lerch.* [31 (10-25-24) 109.]
15. *The behavior at infinity of the gamma and associated functions.* [31 (5-2-25) 484.]
16. *The algebraic structure of the formulas in plane trigonometry.* Three papers. [31 (5-2-25) 484; 32 (9-11-25) 23; 32 (10-31-25) 36.]
17. *A new form of the remainder in the binomial series with applications.* Two papers. [32 (10-31-25) 36–37; 32 (2-27-26) 195.]
18. *The variation of the dielectric constant in the Debye-Hückel theory.* Written with V. K. La Mer. [33 (10-30-26) 12.]

19. *On the convergence region of a power series in several variables.* [34 (4-6-28) 423.]

20. *On the differential equation of the vibrating membrane.* [35 (2-23-29) 293.]

21. *Straight line geodesics in Einstein's parallelism geometry.* Two papers. [35 (2-23-29) 293; 35 (3-29-29) 437.]

22. *Some series expansions in the lattice theory of crystals.* [36 (12-27-29) 186.]

23. *Rotational symmetry in the characteristic functions of the Schrödinger wave equation.* [36 (2-22-30) 218.]

24. *On the theory of potentiometric titration.* [37 (9-8-31) 530.]

LIST OF ABBREVIATIONS

AM = Annals of Mathematics

AMM = American Mathematical Monthly

AMP = Archiv der Mathematik und Physik

ASENS = Annales Scientifiques de l'Ecole Normale Supérieure

BAMS = Bulletin of the American Mathematical Society

BKSVAH = Bihang till Kungliga Svenska Vetenskaps Akademiens Handlingar

CR = Comptes Rendus

JdM = Journal de Mathématiques

JfM = Journal für Mathematik

JPC = Journal of Physical Chemistry

MA = Mathematische Annalen

MZ = Mathematische Zeitschrift

ÖKVAF = Öfversigt af Kungliga Vetenskaps Akademiens Förhandlingar

PNAS = Proceedings of the National Academy of Sciences

PR = Physical Review

PZ = Physikalische Zeitschrift

RCMP = Rendiconti del Circolo Matematico di Palermo

S = Science

TAMS = Transactions of the American Mathematical Society

EINAR HILLE

VOLUME II OF LORIA'S HISTORY

Storia delle Matematiche. By Gino Loria. Volume II. *I Secoli XVI e XVII.* Torino, 1931. 595 pp.

The second volume of Loria's History of Mathematics fulfills the expectations that the excellent first volume aroused.* We have here an account of the early development of modern mathematics, told in an interesting and often illuminating manner, and making full use of the extensive knowledge of the whole field for which Professor Loria is so well known. The book covers the two epoch-making centuries, the sixteenth and seventeenth, extensively and thoroughly. Even a beginner, reading through these stirring narrations, can not fail to be thrilled with the sense of the tingling vitality of those times, and the deep significance of their discoveries for all the long future; while the scholar will admire the aptness with which the work of a man or a period is characterized, and will appreciate, even where he does not always agree with, the originality and vivacity of the author's discussion.

A good idea of the plan and arrangement of the book can be obtained by noting the chapter groupings. They are as follows: Chapter 16, 64 pages on "syncopated algebra"† in Italy (dealing especially with Tartaglia, Cardan, Ferrari, and Bombelli); Chapter 17, 42 pages on syncopated algebra in Europe "Beyond the Alps," with Vieta as the most important name, but giving adequate attention also to Stifel, Recorde, Stevin, and others; Chapter 18, 27 pages on the influence of Humanism on mathematical studies; Chapter 19, 28 pages on trigonometry in the sixteenth century, including an interesting discussion of the quadrature of the circle; Chapter 20, 11 pages on scientific periodicals and societies founded during the seventeenth century; Chapter 21, 36 pages on the "first years of a glorious century," devoted particularly to Napier, Galileo, and Kepler; Chapter 22, 30 pages on the disciples of Galileo, especially Cavalieri and Torricelli; Chapter 23, 23 pages giving accounts of Girard, Harriot, Oughtred, and Hérigone; Chapter 24, 65 pages devoted to Descartes and Fermat; Chapter 25, 33 pages on the revival of pure geometry (the Italian word "Risveglio" in this connection cannot be adequately reproduced in English), the leaders in the movement being of course Desargues and Pascal; Chapter 26, 27 pages devoted to Roberval, Wallis, Barrow, and other immediate precursors of Newton and Leibniz in laying the foundations of the calculus; Chapter 27, called "Intermezzo," 41 pages on miscellaneous geometric discoveries, with Huygens as the most important figure; Chapter 28, 53 pages equally divided between Newton and Leibniz, discussing not only the calculus but also other phases of their mathematical work; Chapter 29, 33 pages on the disciples of Newton and Leibniz, the greatest names being Jacques and Jean

* The first volume was reviewed in this Bulletin, vol. 36 (1930), pp. 336–337.

† This term is explained in Cajori's *History of Mathematics*, New York, 1919, p. 111.

Bernoulli; and Chapter 30, 18 pages on the priority strife between Newton and Leibniz.

With this choice and distribution of material there can not well be any considerable disagreement, as the most important developments, their mutual interactions, and the personalities of the workers, are adequately discussed. The present reviewer would however have preferred that considerably less space—best of all, none at all—should have been given to the quarrels about the solution of the cubic equation, the properties of the cycloid (as to whether Torricelli plagiarized from Roberval and Galileo or not—his complete integrity is now universally acknowledged), and especially the most unfortunate Newton-Leibniz affair. Is it not better that such unprofitable topics be buried in oblivion as speedily as possible? The many pages that would thus be gained might be given to an amplification of many of the topics which now for want of space are rather inadequately treated. For example, Napier's invention of logarithms, Pascal's theorems on the conics, Kepler's and Wallis's methods of "integration," Barrow's ingenious results derived from his "differential triangle," and Newton's studies in algebra and in infinite series. Also, the six pages on "Works of Desargues" could better have given more details of his discoveries than so much about the writings of minor authors who attacked or defended Desargues.

That the style is vivid and picturesque will not need to be mentioned to any one who has read other writings of Professor Loria. If this quality could be recaptured in a translation, it would add a volume of unique value to the small number of excellent histories of mathematics in English.

One detail is somewhat disturbing, that is, the "translation" into Italian of the given names of non-Italian scientists. Such combinations as Biagio Pascal, Isacco Newton, Goffredo Guglielmo Leibniz, to mention only the best known of the names thus treated, seem in inferior taste to the orthography used by the men in question themselves. (Curiously, Lord Brouncker—by oversight?—appears (p. 354) as "William Brouncker.") Misprints, while apparently less numerous than in the first volume, are disturbingly evident, especially in the bibliographies. With few exceptions, however, they are not such as to cause any difficulty to a reader. The date of Desargues's death is 1662, not 1661 (p. 368); and on page 564 there is a reference to "Ostwald's Klassiker der exacten Gesellschaften"! Wallis's "Treatise of Algebra . . . " was published in English in 1685, in Latin in 1693, instead of the dates given on page 463. An index of subjects, to supplement the index of names, would have been a valuable addition.

The appearance of the announced third volume will be awaited with eager anticipation by all who are interested in the history of mathematics.

R. B. McCLENON

SHORTER NOTICES

Analytical Mechanics. By H. M. Dadourian. Third edition. New York, D. Van Nostrand Co., 1931. xiv+427 pp.

The first edition of this well known text, published in 1913, was followed by a second edition in 1915. Although the third edition contains no essential changes, the text has been carefully revised. Some new material has been added and the presentation of a number of topics has been improved.

The author has based his development of the subject upon a single fundamental principle, called the action principle. A statement and explanation of the action principle has been given recently in the American Mathematical Monthly, vol. 38 (1931), pp. 270–274.

W. R. LONGLEY

Die elliptischen Funktionen von Jacobi. By L. M. Milne-Thomson. Berlin, Springer, 1931. xiv+69 pp.

"Five-place tables, with differences, of sn u, cn u, dn u with natural numbers as arguments, arranged according to values of $m(=k^2)$, together with formulas and graphs." The values of the functions are given in natural numbers to facilitate their use with computing machines. There are no other tables of these functions.

H. R. BRAHANA

Sur une Forme plus Restrictive des Relations d'Incertitude. By Louis de Broglie. Paris, Hermann, 1932. 24 pp.

La Projection de Noyaux Atomiques par un Rayonnement très Pénétrant. L'Existence du Neutron. By Irène Curie and F. Joliot. Paris, Hermann, 1932. 22 pp.

These two short books are volumes 31 and 32 in the collection Actualités Scientifiques et Industrielles which is under the general editorship of Louis de Broglie. The first is an analysis, with amplifying notes, of a paper by Landau and Peierls which appeared in the Zeitschrift für Physik, vol. 69. Developing a remark due to Bohr, it is shown that when account is taken of the axioms of the special principle of relativity, Heisenberg's uncertainty principle $\Delta p \cdot \Delta q \geqq h$ must be replaced by $\Delta p \geqq (h/c)\Delta t$, which does not involve the uncertainty in the coordinate measurement q. The application of the uncertainty principle to the measurement of electromagnetic fields is treated. The second book gives an account of experimental observations of a new type of radiation produced when light atoms, such as those of paraffin, are bombarded by α particles. The first experiments of this nature were made by Bothe and Becker in 1930, and they have been repeated by Chadwick, Feather and Dee, and others. The weight of evidence seems to support the hypothesis that the radiation consists of uncharged particles or neutrons. The authors believe that the neutrons effect occasionally the disintegration of the nuclei of atoms into whose sphere of influence they penetrate.

F. D. MURNAGHAN

Mathematics of Relativity; Lecture Notes. By G. Y. Rainich. Ann Arbor, Edwards Brothers, 1932. i+67 pp.

As those who have had the pleasure of hearing Professor Rainich lecture would expect, this set of mimeographed lecture notes is very stimulating, original, and interesting. Vectors and tensors are first introduced in terms of cartesian coordinates in euclidean space of three dimensions, and the tensor form of Euler's hydrodynamical equations and of Maxwell's equations is quickly arrived at. Then follows a chapter on four-dimensional euclidean geometry and an account of an axiomatic, non-coordinate, basis for tensor analysis. This basis, which is largely due to the author, is very important and furnishes a method for the introduction of coordinates. The theory of special relativity with its most important implications is given in Chapter 3. Chapter 4 is devoted to curved space and general (Gaussian) coordinates and an adequate account of the curvature tensor, normal coordinates, etc. is given. Chapter 5 furnishes the best concise account we have seen of general relativity and the three fundamental tests—motion of a planet, bending of a ray of light, and shift of spectral lines. We hope very much that the author will be encouraged to amplify (and supply with adequate references) these notes in a printed volume.

F. D. MURNAGHAN

Lehrbuch der Funktionentheorie. By Ludwig Bieberbach. Volume II, *Moderne Funktionentheorie.* 2d edition. Berlin and Leipzig, B. G. Teubner, 1931. vi+370 pp.

It suffices to compare this treatise with any one published ten years ago or earlier to conceive the tremendous change and progress achieved by the theory of functions of a complex variable during recent years. The questions whose treatment occupied chapters in earlier publications, are given here but few pages, or even are entirely eliminated to be replaced by scores of new problems, methods, and results. The following list of contents hardly can give an adequate idea of the variety of material contained in the *Lehrbuch* of Bieberbach. Chapter 1 (83 pp.) is devoted to the discussion of the classical problem of conformal mapping of simply-connected domains; the interior problem is treated as well as the problem of the correspondence of the boundaries. Brief indications concerning the problem of conformal mapping of multiply-connected domains are given. At the end of the chapter fundamental properties of univalent ("schlichte") functions are exposed, including "Flächensatz," "Verzerrungssatz," and Littlewood's estimates for the coefficients of the power series expansion of a univalent function. Chapter 2 (20 pp.) contains a discussion of indispensable properties of the modular function. Chapter 3 (56 pp.) gives an exposition of fundamental results of the theory of analytic functions bounded in the unit circle, and of its various generalizations. Here we find Schwarz's lemma and its generalizations and modifications (Lindelöf's principle, theorems of Julia and Löwner), Jensen-Nevanlinna's formula and its applications, Schur's method of the determination of the power series expansion of a bounded function, Fatou's theorem, generalizations of Vitali's theorem (theorems of F. and M. Riesz, Khintchine, Ostrowski). Chapter 4 (56 pp.) treats of the uniformisation problem. Problems connected with Picard's theorem are discussed

in Chapter 5. We mention particularly the proof of Bloch, the idea of which begins to play an important role in the most recent investigations. Theory of entire functions is discussed in Chapter 6. In the exposition of the general theory the author uses mainly ideas of R. Nevanlinna. The end of the chapter is devoted to the beautiful recent proof by Ahlfors of the theorem of Denjoy-Ahlfors concerning the asymptotic values of entire functions. Chapter 7 (67 pp.) treats of various problems of analytic continuation and of related questions. We mention theorems of Hadamard, Fabry, Wiegert-Faber, Pólya-Carlson, and many others. The book ends with the very appropriate application of the general theory to the theory of zeta-functions (Chapter 8, 33 pp.).

Several remarks might have been made concerning various details of exposition, a few lapses and mistakes might have been mentioned. In particular it is the reviewer's opinion that the reader will find many difficulties in reading and understanding Chapter 4, on uniformisation, and that this important and difficult branch of function theory is still lacking an adequate exposition. All these remarks will be of comparatively minor importance, however. Most, if not all of the defects will be probably eliminated in the third edition of this extremely useful and suggestive book, which undoubtedly will appear before long.

J. D. TAMARKIN

Vorlesungen über Geometrie. By A. Clebsch and F. Lindemann. Volume I, Part I, Number 3, second augmented edition. Leipzig and Berlin, Teubner, 1932. xvi+101+ii pp.

The portion of the second edition which is at last published in full covers, in 869 pages, the same topics which were treated in 284 pages of the first edition. The principal divisions are:—1. Introductory Considerations, Ranges and Pencils, 2. Curves of the Second Order and Second Class, 3. Introduction to the Theory of Algebraic Forms. The three numbers were issued in an inconvenient manner. Number 1, published in 1906, had pp. 1–480, and broke off in the middle of a chapter on collineations in the ternary domain. Four years later, the publishers put out pp. 481–768, and again stopped *in medias res*, this time in a consideration of sundry problems from the theory of binary forms of higher order. Now, after the lapse of twenty-two years, we have the final pp. 769–869 in a pamphlet which also contains a table of contents and an index for the whole of Part I. It seems unlikely that more of the second edition will ever be published. Since the first, or 1876, edition had 1700 pages, it appears that the second edition covers only about a sixth of the original work.

In the issue now under review Lindemann completes his introduction to the theory of algebraic forms by discussing individual problems which show the connection between binary forms of higher order and the theory of linear differential equations, spherical and Lamé functions. The treatment is by means of extensions of certain theories due to Hilbert. The last two chapters use invariant-theoretic ideas about binary cubic forms, in conjunction with the representation of binary forms in the complex plane, to furnish a general principle which unifies a part of the immense group of theorems about noteworthy points and circles of the triangle. The discussion here is particularly elegant, but it presupposes a considerable knowledge of the geometry of the triangle.

C. A. RUPP

The Taylor Series. An Introduction to the Theory of Functions of a Complex Variable. By P. Dienes. Oxford, The Clarendon Press, 1931. xii+552 pp.

This treatise conducts the reader from the elements of real variable theory into some of the furthest reaches of complex analysis. As we feel that the book will find its chief usefulness in connection with modern function theory, we shall describe first the more advanced chapters.

Chapter VIII deals with Picard's theorem and conformal mapping. By the simple method introduced in 1925 by Bloch, the theorems of Landau and Schottky and Picard's theorem are proved in succession. The Riemann mapping theorem for simply connected regions is proved by the most modern methods.

Chapter IX presents Weierstrass' factorization theorem for integral functions, Mittag-Leffler's theorem, Borel's method for the summation of divergent Taylor series and the representations of analytic functions in star domains, due to Mittag-Leffler and Painlevé.

Chapters X and XI deal with questions inspired, directly or indirectly, by Hadamard's dissertation. One finds here Hadamard's condition for the nonexistence of essential singularities on a circle of convergence and his theorem on the multiplication of singularities. Ostrowski's hyperconvergence is also presented.

Chapter XIII deals with conditions for a power series to represent a bounded function within its circle of convergence (Nevanlinna), with majorants (Hardy, Landau, Bohr), with convergence at singular points (Riesz), and with radial limits (Fatou).

In Chapter XII there is given an extensive treatment of the summation of divergent series. Applications are made in the final chapter (XIV), to the study of the types of divergence of power series at singular points. This last chapter also presents Hadamard's theory of the order of a singular point.

It would be difficult to overestimate the value, for advanced students, of these later chapters in Dienes' book. They reduce to didactic form a large section of the recent literature on complex analysis. These seven chapters, the second volume of Bieberbach's *Funktionentheorie* and Montel's *Fonctions Entières et Méromorphes*, give together quite a complete account of the more modern contributions to complex variable theory.

The first seven chapters of Dienes' book present what is commonly considered elementary complex variable theory, with the exception that, in Chapter V, one finds such topics as Schwarz's lemma, Hadamard's three-circle theorem, and Vitali's theorem. The elementary chapters contain a great amount of excellent material. One might mention the interesting treatment of hypercomplex numbers and also the valuable collections of exercises. However, we do not think it can be claimed that these chapters furnish a sound introduction to function theory. Let us consider, for instance, the question of analysis situs. Chapter VI contains a detailed proof of the Jordan separation theorem. On the other hand, the notion of sense receives no consideration, so that, in Chapter VII, one finds oneself integrating "with the area on the left," on a purely geometric basis, just as in older, frankly intuitive accounts of the subject. The treatment of the number system in Chapter I does not seem to us to be ade-

quate for beginners. In Chapter III, integration is used on the basis of the student's knowledge of the calculus.

It thus appears that, although we have in recent years acquired excellent treatises on higher complex function theory, one must still refer students to the works of Osgood and of Tannery for thorough presentations of the elements of the subject.

Our comments on the earlier chapters of Professor Dienes' book are not to be interpreted as adverse criticism. We have sought merely to indicate what we consider the advantages of this treatise, which is the work of a distinguished authority and which will hold an important place in every mathematical library.

J. F. RITT

Exercices de Calcul Différentiel et Intégral. By E. Lainé. Paris, Vuibert, 1931. 143 pp.

This book contains the solutions of most of the problems given in the written examinations at Paris for candidates "du certificat de Calcul différentiel et intégral" from 1920 to 1930, inclusive. The examinations were held twice each year in June or July and in October and are presented here in chronological order. The solutions are quite complete in themselves and, for the benefit of the students, references for formulas and theory are made to the author's two volume text entitled *Précis d'Analyse Mathématique.*

The ground covered by the examinations and the relative emphasis on each field is indicated by the following distribution. Frequently a problem involves more than one field so that the total number of references is almost double the number of problems. I. Theory of functions of real variables, 25 problems. II. Theory of analytic functions, 9 problems. III. Ordinary differential equations, 19 problems. IV. Differential geometry, 22 problems. V. Partial differential equations, 30 problems.

Each of the sixty-one problems is a very substantial exercise, frequently involving several parts to the question. Professors in American universities will find in this collection some excellent material for use in connection with graduate courses in the fields mentioned above.

W. R. LONGLEY

Exposé Électronique des Lois de l'Électricité. By Marcel Boll. Paris, Hermann, 1932. 72 pp.

The idea of this little book is to give an exposition of the elementary facts of electromagnetism based on the electronic theory. Mathematically there is very little difference between such an account and the older Maxwellian theory with its electric densities, displacement currents, etc., the changes being largely in the symbolism and the physical interpretation. The book is evidently intended mainly for engineers and the mathematics involved is of an elementary character. Thus the equations of Maxwell-Lorentz are not given. There is an interesting chapter of some ten pages devoted to the conduction of electricity through metals.

F. D. MURNAGHAN

Vectorial Mechanics. By Louis Brand. New York, John Wiley and Sons, 1930. xvii+544 pp.

This book has been designed as an introductory text in mechanics for students of engineering and physics. As indicated by the title, the vector method has been used consistently throughout the book. Vector algebra is developed in Chapter I, which is followed by four chapters on statics. Chapter VI gives a brief, but sufficient, exposition of vector calculus. This is followed by a chapter on flexible cables, three chapters on kinematics, four chapters on dynamics, and one on impact.

The pedagogical features of the book are excellent. Careful directions to be observed in solving problems are given, each chapter is followed by a concise summary of the principal results, the theory is fully illustrated by worked examples, and a large collection of problems affords ample material for the development of power and insight. Although the introduction states that "nearly all the answers of the problems are given in a list at the end of the book," the answers are published in a separate pamphlet and will be furnished on the recommendation of the teacher.

Without actual classroom experience the reviewer hesitates to pass judgment on the value of the book as a text for an introductory course. Mechanics is always a difficult subject. Some students grasp the physical concepts but have difficulty with the mathematics. Others have most trouble with the physical ideas. It may well be that the condensed vector notation will be an aid to both types. On the other hand the dot product and cross product may only add to the difficulty of grasping the subject. There seems to be no doubt, however, that the book will appeal strongly to those who have some familiarity with the subject, particularly to students described in the preface as those "who, freed from the necessity of making grades, wish to review mechanics thoughtfully and thoroughly, beyond the point of having a few pat rules for solving special types of problems."

W. R. LONGLEY

Théorème de Fermat, son Histoire. By R. Noguès. Paris, Librairie Vuibert, 1932. 179 pp.

The book consists of two parts. The first part is mainly historical, citing in chronological order a number of papers on Fermat's Last Theorem, and including methods employed and results obtained. In the second part, the articles mentioned in the first part are again referred to and details of the proofs of the results are, to some extent, given. The first chapter is devoted to details concerning Fermat's life and his mathematical work in general, as well as to his statements concerning the Last Theorem. The last article mentioned in the book has the publication date of October 12, 1931.

The author has included in his account a number of false proofs of the theorem, in some cases without comment of his own. On the other hand he has omitted reports on many publications which have generally been regarded, by those particularly interested in Fermat's Last Theorem, as being among the principal contributions which have been made to our knowledge concerning the problem. For example, he treats in detail Kummer's article of 1850 concern-

ing the theorem, but Kummer's paper of 1857 is not even listed, although some of the results are mentioned in connection with Noguès' report on the contents of Mordell's pamphlet of 1921, (French translation, 1929).

Chapter XI was designed to cover the period from 1910–1931, but among the nine reports on articles given therein all but two of the authors are French! No mention is made of Frobenius' article of 1914, (except in the report on Mordell), or of the interesting results obtained by Furtwängler, Pollaczek, Beeger, or Morishima, during the period treated.

It appears that little if any use was made by Noguès in the preparation of his book of the *Report on Algebraic Numbers*, II, Bulletin of the National Research Council, 1928, by G. E. Wahlin and the present writer, which treats the class number of an algebraic field and Fermat's Last Theorem.

I have already referred to the fact that incorrect proofs of the Last Theorem have been reproduced by our author in some cases without comment by him. With one exception, this should cause little difficulty to the reader, as the errors may be readily detected. In the case of the report on *Démonstration du Théorème de Fermat*, by E. Fabry, (1913), however, the error in his supposed proof that the Fermat relation is impossible in Case I, lies considerably below the surface. It was pointed out by Mirimanoff in the Comptes Rendus, vol. 157 (1913), pp. 491–492.

The principal value of the book, in my opinion, lies in the fact that the author has given extensive accounts of the work of Legendre, Dirichlet, Lamé, V. Lebesgue, and Mirimanoff.

H. S. VANDIVER

Les Principes de la Mécanique Quantique. By P. A. M. Dirac. Traduit par Al. Proca et J. Ullmo. Paris, Les Presses Universitaires de France, 1931. viii +314 pp.

This translation of Dirac's *Principles of Quantum Mechanics* appears as Vol. 21 of the series *Conférences—Rapports de Documentation sur la Physique.* The English edition was amply reviewed in this Bulletin, vol. 37 (1931), pp. 495–496, and it is only necessary to say here that the French edition contains a useful appendix of nine pages by one of the translators (Al. Proca) giving a summary account of the most important properties of the Poisson and Lagrange brackets of classical mechanics. The translation has been carefully done, and the printing and general appearance of the book are admirable.

F. D. MURNAGHAN

NOTES

In addition to those of the editorial staff of the Bulletin, the following persons have assisted the editors, either by refereeing papers or by advising concerning papers offered for publication in the present volume: C. R. Adams, A. A. Albert, W. L. Ayres, E. T. Bell, B. A. Bernstein, A. B. Brown, Alonzo Church, Arnold Dresden, G. C. Evans, Raymond Garver, D. C. Gillespie, E. V. Huntington, Louis Ingold, Dunham Jackson, O. D. Kellogg, A. J. Kempner, E. P. Lane, D. H. Lehmer, M. I. Logsdon, C. C. MacDuffee, A. D. Michal, C. N. Moore, R. L. Moore, Oystein Ore, F. R. Sharpe, Virgil Snyder, E. B. Stouffer, J. D. Tamarkin, H. S. Wall, J. H. M. Wedderburn, H. S. White, Hassler Whitney, G. T. Whyburn, W. M. Whyburn, R. L. Wilder. The editors desire publicly to recognize this service.

In addition to the members of the editorial board of the Transactions of this Society, the following persons have assisted the editors by refereeing papers offerred for publication in the volume for 1932: A. A. Albert, W. L. Ayres, Alonzo Church, L. E. Dickson, Jesse Douglas, H. M. Gehman, J. J. Gergen, L.M. Graves, T. H. Hildebrandt, E. Hopf, E. V. Huntington, B. O. Koopman, C. C. MacDuffee, E. D. Pepper, Tibor Radó, Stanislaw Saks, J. B. Shaw, H. S. Vandiver, J. H. M. Wedderburn, J. H. C. Whitehead, Hassler Whitney, D. V. Widder. The editors appreciate this assistance and desire to give it this public recognition.

Three volumes of the Colloquium Publications of this Society have appeared in 1932 (volumes 13, 14, and 15 of the series): *Foundations of Point Set Theory*, by R. L. Moore; *Differential Equations from the Algebraic Standpoint*, by J. F. Ritt; *Linear Transformations in Hilbert Space and their Applications to Analysis*, by M. H. Stone.

Copies of Nazimoff's *Applications of the Theory of Elliptic Functions to the Theory of Numbers*, translated from the Russian and mimeographed (see this Bulletin, vol. 35, p. 735), may now be had for $1.50.

The Astrophysical Union met at Cambridge, Mass., September 2–9, 1932. On this occasion Sir Arthur Eddington delivered an address on *The expanding universe.*

Dr. Max Planck, professor emeritus of theoretical physics at the University of Berlin, has recently received a Goethe medal from the German government.

Assistant Professor W. L. Ayres has received the Henry Russell Award for 1932 at the University of Michigan. This prize is awarded annually to a young man of the faculty of that University for past achievement and future promise in research. The prize was established in 1925.

A monument to P. S. Laplace was unveiled at his birthplace (Beaumont-en-Auge), on August 13, 1932.

The University of Paris has conferred an honorary doctorate on Sir C. V. Raman, professor of physics at the University of Calcutta.

The University of Belfast has conferred an honorary doctorate of science on Dr. A. C. Dixon, professor of mathematics at Queen's University, Belfast.

Dr. William Bowie, of the United States Coast and Geodetic Survey, has been elected a member of the State Russian Geographical Society.

Professor A. H. Compton, of the University of Chicago, has been elected a corresponding member of the Prussian Academy of Sciences.

Dr. Alois Kovarik, professor of physics at Yale University, was the recipient on July 2 of an honorary Doctor of Science from Charles University in Prague, and on June 10 of the Memorial Medal from Comenius University at Bratislavia.

Dr. Irving Langmuir, of the General Electric Company, has been elected a member of the Academy of Sciences of Halle.

Dr. Louis de Broglie, professor "sans chaire" at the University of Paris, has been appointed to the professorship of physical theories at that university.

Sir C. V. Raman, professor of physics at the University of Calcutta, has been appointed director of the Indian Institute of Science, Bangalore.

Dr. Robert Sauer, of the Aachen Technical School, has been promoted to an associate professorship of descriptive geometry and applied mathematics.

Associate Professor Otto Volk has been promoted to a professorship of mathematics at the University of Würzburg.

The University of Oregon and Oregon State Agricultural College have been put under one board and one chancellor, and a number of changes in function have been made. One of these is to place all advanced work in science and mathematics at the State College, and Professor W. E. Milne, who has been professor of mathematics at the University since 1919, has been made head of the department of mathematics at the State College.

The Institute of Advanced Study, founded by Louis Bamberger and Mrs. Felix Fuld, and directed by Dr. Abraham Flexner, will open its mathematics school, the first of a series of schools for research, in the fall of 1933, using Fine Hall, Princeton University, as its temporary headquarters. Professor Albert Einstein, of Berlin, has been appointed professor of mathematical physics, Professor Oswald Veblen professor of mathematics, Dr. Walter Mayer associate in mathematics, and Dr. J. L. Vanderslice assistant.

Dr. Jan H. Oort, lecturer and observer at the University of Leyden, arrived in Delaware, Ohio, October 4, to begin photometric studies of nebulae in a cooperative plan with the Perkins Observatory of Ohio Wesleyan University. While at the Perkins Observatory Dr. Oort will give a special series of lectures on stellar statistics on Thursday afternoons, beginning October 13. The titles of the lectures are: "The Observational Data," "Methods of Determining Stellar Distributions," "General Structure of the System of Stars," "Stellar Dynamics I," "Stellar Dynamics II," and "Galactic Rotations." The lectures and discussions are open to all interested.

Professor H. Weyl of the University of Göttingen has accepted the invitation of Swarthmore College to give the William J. Cooper Foundation Lectures for 1932–33. He is expected to be there from the middle of February until the middle of March. During this period, he is to give five lectures on the general subject of Mind and Nature and to take part in the honors work of the college. The lectures are open to the public.

Dr. O. E. Brown has been promoted to an assistant professorship at the Case School of Applied Science.

Mr. R. C. Bullock, of the University of North Carolina, has been appointed to an acting professorship at Lambuth College, Jackson, Tenn.

Dr. Fay Farnum has been promoted to an assistant professorship at Washington Square College, New York University.

Assistant Professor P. H. Graham, of Washington Square College, New York University, has been promoted to a professorship of mathematics.

Professor Alfred Hume, of the University of Tennessee, has been appointed chancellor of the University of Mississippi.

Dr. Eberhard Hopf has been appointed assistant professor of mathematics at the Massachusetts Institute of Technology.

Assistant Professor F. W. John has been promoted to an associate professorship at Washington Square College, New York University.

Assistant Professor A. L. Meder, of the department of mathematics at the New Jersey College for Women, has been appointed acting dean of that college.

Professor O. M. Norlie, of Hartwick College, has been appointed director of the new graduate school of Hartwick Seminary, Brooklyn.

Dr. J. H. Service, of Henderson State College, has been appointed assistant professor of mathematics at Michigan College of Mining and Technology.

Dr. H. L. Slobin, head of the department of mathematics at the University of New Hampshire, has been appointed dean of the Graduate School.

Dr. W. R. Thompson has been appointed research assistant in pathology at Yale University.

Assistant Professor Marion M. Torrey, of Goucher College, has been promoted to an associate professorship.

Dr. Joseph Wilczewski, of St. Louis University, has been appointed to a professorship at Mount St. Michael.

Miss Marguerite L. Zeigel has been appointed to a position in the department of mathematics and physics at Lander College, Greenwood, S. C.

The following appointments to instructorships in mathematics are announced:

Long Beach (California) Junior College, Mr. A. L. Buckman;

College of the City of New York, Dr. E. L. Post;

New York University, University Heights Branch, Mr. F. C. Hall;

Michigan College of Mining and Technology, Mr. Nels Johnson;

Tulane University, Mr. J. F. Thomson.

Professor Max Wolf, director of the Königstuhl Observatory of Heidelberg, died October 3, 1932, at the age of sixty-nine.

Professor W. M. Brodie, of the Virginia Polytechnic Institute, died in April, 1932.

Mr. R. L. Faris, hydrographer and geodetic engineer, assistant director of the United States Coast and Geodetic Survey, died October 5, 1932, at the age of sixty-four.

The death is reported of Dean H. J. Hughes, of the Engineering School of Harvard University.

M. B. Snyder, formerly director of the Philadelphia Observatory and professor of mathematics at the Central High School, Philadelphia, died September 27, 1932, at the age of eighty-four. Professor Snyder had been a member of the American Mathematical Society since 1895.

The death is reported of Professor G. F. Swain, of Harvard University.

Professor E. J. Wilczynski, emeritus professor of mathematics at the University of Chicago, died September 15, 1932, at the age of fifty-six. Professor Wilczynski had been a member of the American Mathematical Society since 1899. A more adequate notice will appear in an early issue of this Bulletin.

ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THIS SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross-references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

231. Miss Beatrice Aitchison: *Concerning the mapping of locally connected continua on to simple arcs.* Preliminary communication.

Mazurkiewicz has determined a necessary and sufficient condition to be satisfied by a dendrite D, so that there exists a real continuous function over D, taking any value only a finite number of times at most and transforming D into a simple arc (Fundamenta Mathematicae, vol. 18). Results already obtained indicate that, by methods being developed in the present paper, based on the cyclic element structure of locally connected continua, it will be possible to find a sufficient condition for such a continuum C to be transformed by a similar function into a simple arc. If the space C has a countable number of end points it can be transformed by a function of this type into a dendrite D, which satisfies a part of Mazurkiewicz's conditions. If under known suitable conditions on C, D can be shown to satisfy all of these conditions, and hence can be transformed into a simple arc in this definite way, the space C can also be transformed into a simple arc by this special type of function. (Received September 2, 1932.)

232. Dr. E. Sperner: *On topological transformations of a plane without invariant points.*

In the first part of this paper new and simple proofs are given for the theorems of L. E. J. Brouwer concerning path curves and transformation fields of sense-preserving topological transformations of a plane without invariant points. In the second part necessary and sufficient conditions are derived for a topological transformation of the plane to be topologically equivalent to a translation. (Received September 2, 1932.)

233. Professor H. P. Robertson: *Possibilities for a universe with non-negative pressure.*

A survey is made of the long-range behavior of all spatially homogeneous and isotropic space-times suitable for relativistic cosmology subject to the restriction $p \geqq 0$. (Received August, 30, 1932.)

234. Professor A. D. Michal and Mr. J. L. Botsford: *Normal representations and linear connections.*

In a preceding paper the authors considered a theory of differential invariants of an affine connection and a linear connection with the aid of "geodesic representations of order r" (see abstract 38-9-220, this Bulletin, and Proceedings of the National Academy, August, 1932). In the present paper the properties of "normal representations" are used to develop a slightly different approach to the subject. (Received September 1, 1932.)

235. Professor A. D. Michal and Dr. R. S. Martin: *A set of expansions in vector space.*

The authors consider a complete linear vector space S closed under multiplication by elements of A, the real or complex number system. There is postulated a bilinear operation of composition such that the elements of S form a ring with respect to addition and composition. A sequence of abstract polynomials is defined by means of the recursion formulas $B_n(T)=a_{n-1}(T)T-B_{n-1}(T)T$; $a_n(T)=[B_n(T)]/n$ where $[\cdots]$ is a linear continuous operation on S to A which is called contraction and has the property $[T_1T_2]=[T_2T_1]$. The properties of the functions $D(T)\equiv\sum_0^\infty a_n(T)$ and $B(T)\equiv\sum_1^\infty B_n(T)$ are considered. Among the results are, for example, that B and D are analytic and Fréchet differentiable functions of T for $\|T\|<1$ ("analytic" is used in the sense previously defined by one of the authors; see R. S. Martin, California Institute, thesis, 1932), and that $D(T)$ does not vanish for $\|T\|<1$. (Received August 31, 1932.)

236. Professor G. C. Evans: *Note on the gradient of the Green's function.*

Let T be a bounded, open, simply-connected region in the plane or in space, $g(M, P)$ its Green's function with pole at M, and let $I=\int_T|\nabla g(M, P)|dp$. In the plane, it is shown by means of the inequalities of Schwarz and Hardy, using complex variables, that I is bounded independently of the position of M; in fact, $I\leqq(2\pi Tg)^{1/2}(1-e^{-g})^{-1}$, where g is an arbitrary positive number. The minimum value of this expression is 2.22 $(\pi T)^{1/2}$, whereas for the circular region of measure T, $I=2(\pi T)^{1/2}$. The length of the level curve $g(M, P)=g$ is $\leqq(2\pi Tg)^{1/2}(e^g-1)^{-1}\leqq(2\pi T/g)^{1/2}$. In space, the subset F, from each point of which T is star-shaped, is null or is closed with respect to T. Making use of Gergen's lower bound for the absolute value of the gradient of the Green's function for a star-shaped region (American Journal of Mathematics, vol. 53 (1931), pp. 746-752), it may be shown that for all points of F, $I\leqq4\pi g^{-1}+(4\pi gT)^{1/2}$, g any positive number. In particular, $I\leqq6.35(\pi^2T)^{1/3}$. If T is convex, of course $F=T$. (Received August 12, 1932.)

237. Professor J. H. Binney: *An elliptic system of integral equations on summable functions.*

The functions $\phi=D_x\psi+(2\pi)^{-1}\int_T r^{-1}(\cos(r, y)dB(e)-\cos(r, x)dA(e))$, $\theta=D_x\psi+(2\pi)^{-1}\int_T r^{-1}(\cos(r, x)dB(e)+\cos(r, y)dA(e))$ where ψ is an arbitrary

solution of Laplace's equation in the simply connected plane region T, and $A(e)$, $B(e)$ are additive functions of point sets in T, form a system of solutions of the pair of integral equations $\int_s \phi dy + \theta dx = A(s)$ and $\int_s -\theta dy + \phi dx = B(s)$, where $A(s)$ and $B(s)$ are functions of curves of limited variation corresponding to the functions of point sets $A(e)$ and $B(e)$ respectively, for every curve s in T for which $\int_T r^{-1} |dA(e)|$ and $\int_T r^{-1} |B(e)|$ represent summable functions on s; in particular for almost all rectangles in T. Conversely, if ϕ and θ are summable superficially in T and are solutions of the pair of integral equations on almost all rectangles, they may, except for removable discontinuities at most on a set of superficial measure zero, be expressed in the form given above. (Received August 12, 1932.)

238. Professor B. A. Bernstein: *On Nicod's reduction in the number of primitives of logic.*

Nicod, using Sheffer's "stroke" operation, obtained a drastic reduction in the number of primitives for the theory of "elementary" propositions in *Principia Mathematica*. The reduction consists in deriving eight of the ten *Principia* primitive propositions from a set of three postulates. But Nicod does not discuss the consistency or the independence of his postulates; fails to consider the *Principia* primitives (two propositions and two ideas) not involved in the above eight propositions, and fails to prove that his postulates are derivable from the *Principia* primitives. The author supplies Nicod's omissions. He shows that Nicod's postulates, when rewritten in customary mathematical language, are consistent and independent; that the four *Principia* primitives ignored by Nicod are redundant; and that Nicod's postulates are derivable from those of the *Principia*. In connection with the last item is pointed out the fact that if a mere derivation of a postulate set S from a smaller set S' constitute a reduction of S, then a reduction in the number of the *Principia* primitives exists that is far more simple and far more drastic than is the Nicod reduction. (Received August 18, 1932.)

239. Professor A. D. Campbell: *Note on cubic surfaces in the Galois fields of order* 2^n.

In this paper a study is made of the cubic surfaces in the Galois fields of order 2^n. The Hessian of any such surface vanishes identically. For certain cubic surfaces the first polar of every point in space is a double plane. The twenty-seven straight lines on a cubic are obtained by a new short method. There turn out to be four, three, two, one or no double points on a cubic. The equation of the cubic in plane coordinates is written down and is found to be of the twelfth degree or of the ninth degree. (Received September 16, 1932.)

240. Dr. I. O. Horsfall: *Transformations associated with the lines of a cubic, quadratic, or linear complex.*

Two equations, $\sum x_i f_i(p) = 0$ and $\sum x_i F_i(p) = 0$ $(i = 1, 2, 3, 4)$, bilinear in x_i and the line coordinates p_{ik}, define an extensive type of cubic complex and also map the complex on the space (x) so that each line (l) is mapped by a point

(x) on (l). The cubic complex of lines joining corresponding points of the general cubic involutorial transformation is included as a special case. The method is also applied to two known cases of the quadratic complex and the linear non-special and special complex. (Received September 6, 1932.)

241. Mr. George Comenetz: *Translation families of heat curves.*

A flow of heat by conduction in the plane defines a family of *heat curves*: the ∞^2 isothermals throughout the flow. When has such a *heat family* a specially simple geometrical structure? (See Kasner, *Geometry of the heat equation*: First paper, Proceedings of the National Academy of Sciences, vol. 18 (1932), p. 475.) We discuss the problem of determining the heat families which consist merely of the ∞^2 copies by translation of a single curve. In a special case we find explicit results for the equations of the families in terms of elementary functions. (Received October 8, 1932.)

242. Mr. Aaron Fialkow: *The geometry of degenerate heat families.*

Professor Kasner has defined "heat family" and "degenerate heat family" in a paper published in the Proceedings of the National Academy of Sciences, vol. 18 (1932), No. 6. In this paper, he has shown that there are three distinct types of degenerate heat families defined by the equations of Laplace, $\Delta u=0$, Poisson (special), $\Delta u=1$, and Helmholtz-Pockels, $\Delta u=\pm u$. A geometric characterization of the Laplace type is known. In his seminar of 1917, Kasner indicated that the other two types must each possess two fourth-order geometric properties. In our work, we derive these two properties of each type, and furthermore prove that they are sufficient to characterize these types. One geometric equation, $\gamma_1(d^2\gamma/ds^2-d^2\gamma_1/dn_1ds_1)+(d\gamma_1/dn_1-\gamma_1^2)(d\gamma/ds+d\gamma_1/ds_1)=0$, which is common to both types, is shown to characterize completely the more general equation $\Delta u=f(u)$. (Received October 6, 1932.)

243. Professor A. K. Mitchell: *On teleparallelism in Finsler space.* Preliminary report.

Synge, Taylor, and Berwald have considered parallel displacement of tensors in Finsler space and have derived equations corresponding to those of Levi-Civita for parallel transport in Riemannian space. In the present paper the equations of transformation, satisfied by the coefficients of connection used by Synge and Taylor, are considered. A definition is made of coefficients, for the Finsler space, which correspond to those used by Einstein in his equations of teleparallelism and which lead to equations of parallelism for Finsler space analogous to those, for Riemannian space, used by Einstein in his unified field theory. (Received September 20, 1932.)

244. Dr. Leo Zippin (National Research Fellow): *Characterization of the closed 2-cell.*

We give the following topologic characterization of the closed 2-cell (a set of points homeomorphic with a plane circle plus its interior): the closed 2-cell

is a compact continuous curve C containing a simple closed curve J and at least one arc spanning J (this shall mean an arc having its end points and these only on J) *such that every arc spanning J irreducibly separates C.* (Received September 27, 1932.)

245. Professor W. A. Wilson: *On unicoherency about a simple closed curve.*

If a metric space Z contains a simple closed curve J, we say that Z is unicoherent about J if, for every division of J into open arcs α and β by points a and b and for every decomposition of Z into closed sets P and Q such that $\alpha \cdot Q = \beta \cdot P = 0$, some component of $P \cdot Q$ joins a and b. This paper discusses some general properties of this extension of the idea of unicoherent continuum, including several equivalent definitions of the extension. It is shown that unicoherency about every simple closed curve contained in the space is the necessary and sufficient condition for a compact metric continuum to be a unicoherent continuum, that every compact metric space Z unicoherent about a simple closed curve J contains a set P irreducible with respect to the property of being closed and unicoherent about J, and finally that, if Z is a locally connected compact metric continuum irreducible with respect to the property of being closed and unicoherent about a simple closed curve J and every simple arc ab such that $(ab) \cdot J = a + b$ disconnects Z, then Z is the homeomorphic image of a two-dimensional simplex. (Received September 23, 1932.)

246. Mr. Victor Perlo: *On the distribution of Student's ratio for samples from non-normal populations.*

We present the exact sampling distribution of Student's ratio, t, comparing sample mean with sample standard deviation, for samples of three drawn from a rectangular population; also the probability P that t will exceed a given value. Letting p be the value corresponding to P for samples drawn from a normal population, for which the distribution of t, i.e., Student's distribution, is known for samples of n, we also obtain the limit of the ratio P/p as t approaches infinity, for samples of n. Since this limit is the upper bound of the ratio P/p for finite values of t, we have a limit within which we may use Student's distribution to calculate P. Then we show that the limiting ratio P/p for samples drawn from rectangular distributions is the upper bound of the corresponding limiting ratio for samples drawn from any unimodal symmetrical distribution, and with a suitable correcting factor, for any unimodal distribution, indicating the use of the limit of P/p, suitably corrected for particular curves, is the bounding factor for the use of Student's distribution for samples drawn from non-normal populations. (Received October 8, 1932.)

247. Professor B. H. Camp: *The converse of Spearman's two-factor theorem.*

There have been several attempts to prove, or to disprove, the converse of Spearman's two-factor theorem. As shown by Irwin, these various methods of proof result ultimately in the same expression for the so-called general factor.

Although the several proofs are necessarily alike in many respects, the different authors appear to have different pictures in mind at the background of their analytical demonstrations. The same may be said of the proof presented in this paper. In addition, I have inserted a certain necessary but hitherto neglected hypothesis, have investigated the possibility of the use of other than linear functions as the basis of the formation of the general factor function, have given a numerical example in which this factor is not unique, and have discussed more fully the important additional question raised by Piaggio as to whether this factor is "almost" unique. (Received September 29, 1932.)

248. Mr. Archie Blake: *Canonical expressions in Boolean algebra.*

In a finite Boolean algebra various canonical expressions can be defined, including the well known *developed* form and a new *simplified* form. These forms always exist uniquely, and the simplified form has the further important property of being the simplest expression that includes formally all terms that it includes at all. Constructions are given for the simplified form, and applications are indicated. (Received September 7, 1932.)

249. Dr. K. E. Rosinger: *A new theorem in combinatorial analysis with application to a classification of triadic relations.*

Given a K of n distinct elements, $n \geqq 3$; let m represent the number of triads formed from the elements of K such that (a) each triad consists of distinct elements, (b) no two triads have more than one element in common, (c) the order of elements within the triad is irrelevant. Then $m \leqq (n^2-n)/6$. By means of the theorem just stated, it is proved that if a triadic relation *Rabc* is such that it implies every permutation of its terms, the relation between the variables and their values must remain constant. Graphically, this means that the logical matrix can be rotated arbitrarily without changing its significance only with reference to a fixed set of coordinates. The proof utilizes the following two lemmas. (I) If $(n^2-n)/6$ is an integer, then $n \neq 3x+2$, $x \geqq 0$. (II) If $n \neq 3x+2$, $x \geqq 0$, then $n^2-n \equiv 0$, mod 6. (Received October 5, 1932.)

250. Mr. E. H. Hadlock: *On the progressions associated with a ternary quadratic form.*

H. J. S. Smith (Collected Mathematical Papers, vol. 1, p. 470) has proved that there exists a primitive ternary quadratic form if and only if the generic characters associated with the form satisfy one of three formulas according as f is properly primitive, f is improperly primitive, or F is improperly primitive. The purpose of this paper is to express these formulas in terms of the progressions associated with the form (see B. W. Jones, *A new definition of genus for ternary quadratic forms*, Transactions of this Society, vol. 33 (1931), pp. 92–110). This leads to the fact that the number of sets of progressions of a certain kind is odd or even according as f is positive or indefinite. It is also found that with every positive form there are associated infinitely many progressions of numbers not represented by f. (Received October 3, 1932.)

251. Dr. S. S. Cairns: *The generalized theorem of Stokes.*

Using the theorem of abstract 38–11–257, we prove the generalized Theorem of Stokes for an orientable regular bounded topological n-manifold M_n in $(n+r)$-space. We use the method employed in 3-space by O. D. Kellogg (*Foundations of Potential Theory*, chapter IV). Let $(y_1, \cdots, y_{n+r})$ be rectangular cartesian coordinates in our $(n+r)$-space. Consider a set of $\binom{n+r}{n-1}$ functions, $Y_{i_1 \cdots i_{n-1}}$, of $(y_1, \cdots, y_{n+r})$, defined and of class C' on M_n, where the i's are numbers of the set $(1, \cdots, n+r)$ arranged in order of increasing magnitude. Using generalized direction cosines, $(\gamma_{i_1 \cdots i_n})$ (abstract 38–3–114), of the tangent n-space to M_n at any point, and also $(\beta_{i_1 \cdots i_{n-1}})$, of the tangent $(n-1)$-space to the boundary, B_{n-1}, of M_n, we formulate the theorem thus: $\int_{M_n} \sum D_{i_1 \cdots i_n} \gamma_{i_1 \cdots i_n} d\tau = \int_{B_{n-1}} \pm \sum Y_{i_1 \cdots i_{n-1}} \beta_{i_1 \cdots i_{n-1}} d\sigma$, where the summations are over all the sets $(j_1 \cdots j_n)$ and $(i_1 \cdots i_{n-1})$, respectively, and where $D_{i_1 \cdots i_n} = \sum_{k=1}^{n} (-1)^{k+1} (\partial / \partial y_{j_k}) (Y_{j_1 \cdots j_{k-1} j_{k+1} \cdots j_n})$. The alternative signs are given because a change of orientation on M_n (or B_{n-1}) alters the signs of all the γ's (or all the β's). This is equivalent to the usual formulation, but has the advantage of being formally independent of any parametric representation of M_n. (Received September 7, 1932.)

252. Mr. J. G. Deutsch: *On the existence and equality of repeated Riemann integrals.*

In this paper, a necessary and sufficient condition is determined for the existence of the repeated Riemann integrals of a function of two variables. A necessary and sufficient condition that these integrals not only exist but be equal is also established. These conditions are of the nature of simple comparison tests and specialize to yield the results of Young, Lichtenstein, and Gillespie. (Received October 6, 1932.)

253. Dr. W. J. Trjitzinsky: *Analytic theory of linear q-difference equations.*

In the past the analytic theory of linear q-difference equations has been developed under the assumption of restricted roots of the characteristic equation (Carmichael, Birkhoff). In the present paper the author develops the analytic theory for the general case of unrestricted roots. The main results are embodied in the Fundamental Existence Theorem. It is proved that in all cases there exist complete sets of analytic solutions with the asymptotic form (in certain regions) of the formal series solutions. This is proved on the basis of a certain method of contour q-difference summation. The importance of this paper, in the theory of q-difference equations, is analogous to that of the joint paper by G. D. Birkhoff and the author in the theory of difference equations (Acta Mathematica, vol. 59 (1932)). The present paper will appear in the Acta Mathematica. (Received October 6, 1932.)

254. Dr. Wladimir Seidel: *On the distribution of values of bounded analytic functions.*

If $f(z)$ is a bounded analytic function in the unit circle $|z| < 1$, the limit $\lim_{r \to 1} f(re^{i\theta}) = f^*(e^{i\theta})$, $z = re^{i\theta}$, exists by a theorem of Fatou for all values of θ in

the interval $0 \leq \theta \leq 2\pi$ save perhaps in a set of measure zero. This paper principally deals with the case when $|f^*(e^{i\theta})| = 1$ for almost all points of either the whole circumference or an arc of the circle $|z| = 1$. Among the theorems proved is the following: *If $f(z)$ is a bounded analytic function in the circle $|z| < 1$ such that $|f^*(e^{i\theta})| = 1$ for almost all points of the circumference $|z| = 1$, and if it omits a value α, $|\alpha| < 1$, in the circle $|z| < 1$, then there exists a radius on which $f(z)$ approaches the value α.* By means of this theorem we can study the conformal representation of certain types of multiply-connected regions on a circle, the representation being defined by Blaschke products. The following extension of Schwarz's reflection principle is also obtained: *Let $w = f(z)$ be a bounded analytic function in the unit circle $|z| < 1$: $|f(z)| < 1$. If $|f^*(e^{i\theta})| = 1$ for almost all points of an arc A of the circle $|z| = 1$ and if $f(z)$ does not assume in the unit circle $|z| < 1$ values which form a set of positive superficial measure in the circle $|w| < 1$, then $f(z)$ may be continued analytically beyond the arc A in accordance with the relation $f(1/\bar{z}) = 1/\overline{f(z)}$.* (Received October 8, 1932.)

255. Mr. L. A. MacColl: *On the distributions of the zeros of certain analytic functions.*

The subject of this paper is the distribution of the zeros of a function of the form $f(z) = \sum_{j=0}^{J} \exp(\lambda_{jN} z^N + \cdots + \lambda_{j1} z + \lambda_{j0})$, where J and N are positive integers, and the λ's are constants. The problem is a generalization of one studied by Wilder, Tamarkin, Pólya, Schwengler, and others. The λ's determine a set of critical rays in the z-plane; these are classified as primary, secondary, etc. When $N = 1$ only primary critical rays occur. Each zero of $f(z)$ is a point of one or more of a set of half-strips, which are equal in number to the critical rays, and each of which extends in the direction of a different one of these rays. Asymptotic expressions are given for the numbers of zeros in the various half-strips and within the large circle $|z| = r$. Whereas when $N = 1$ these expressions are the same, to within constant factors, for the several half-strips, in the general case this is not so. (Received October 4, 1932.)

256. Dr. Selby Robinson (National Research Fellow): *Properties related to the Borel property and to the closure of derived sets.*

It is shown that in a space having properties B and D of Hausdorff and in which every point is interior to a set having the property of Borel-Lebesgue, the interior of every set is open. We have theorems on the relations of various sorts of reducibility of coverings to each other and to the closure of sets. In terms of set functions introduced by Chittenden, we give two necessary and sufficient conditions that the interior of every set in a space (P, K) shall be open. If a space (P, K) has the latter property, the points of condensation of any set form a closed set. (Received October 10, 1932.)

257. Dr. S. S. Cairns: *The triangulation of regular topological n-manifolds in $(n+r)$-space.*

The theorem below extends and amplifies a previously announced result (National Academy Proceedings, vol. 16 (1930), pp. 488–491, Theorem 2).

For *n-spread*, the term *topological n-manifold* is substituted. A *regular bounded topological n-manifold* means a compact connected n-dimensional point set, M_n, made up of (1) a set M_n' of points each of which has a regular n-cell (loc. cit.) for one of its neighborhoods on M_n, and (2) a set B_{n-1} of limit points of M_n' none of which has such a neighborhood on M_n, where B_{n-1} is the sum of a finite number of distinct regular unbounded topological $(n-1)$-manifolds. Theorem: A regular topological n-manifold (bounded or unbounded) in euclidean $(n+r)$-space is coincident with a simplicial n-complex. The vertices of each i-cell ($i=1, \cdots, n$) of the complex determine an i-simplex in the $(n+r)$-space. The set of all simplexes so determined is a simplicial polyhedron, whose approximation to the given manifold can be made arbitrarily close. (Received September 7, 1932.)

258. Dr. Hassler Whitney (National Research Fellow): *Cross sections of curves in 3-space.*

Let F be a regular family of curves (see National Academy Proceedings, March, 1932, pp. 275–278) filling a region in euclidean 3-space. Using a theorem by the author (to appear in the Transactions of this Society; see abstract 37–11–354) it is shown that there is a cross section through any point of the region which is a closed 2-cell. Hence the curves in the neighborhood of a point are equivalent to a family of parallel straight lines. (Received October 7, 1932.)

259. Dr. Gordon Pall: *The structure of the number of representations function in a binary quadratic form.*

To extend the results of a recent paper (abstract No. 37–5–229) to any integral binary quadratic form of non-zero discriminant, there are studied fully properties of sets of representations (abstract No. 38–1–25) in a binary quadratic form, that is, representations equivalent through automorphic transformations. In particular the problem of representing a number by trial in an indefinite form is simplified. (Received October 10, 1932.)

260. Dr. S. B. Littauer and Professor Marston Morse: *A characterization of fields in the calculus of variations.*

The present paper is concerned with a geometric characterization of conjugate points and focal points in the calculus of variations. In the plane one can prove that conjugate points of a point P on an extremal g are the points on g whose neighborhoods are not covered in a one-to-one manner by the family of extremals issuing from P with directions near that of g at P. The methods used to prove this theorem in the plane are inadequate if $n>2$. We are able, however, to prove the following theorem: *In the analytic case, a necessary and sufficient condition that a point Q on an extremal g be a conjugate point of a point $P(\neq Q)$ on g is that the family of extremals through P issuing from P with directions near that of g at P fail to cover the neighborhood of Q in a one-to-one manner.* In the proof of this theorem no assumption is made as to the nature of the envelope, and herein lies the power as well as the difficulty of the theorem. The sufficiency of the condition is of course well known. The theorem

is of particular interest because the vanishing of the Jacobian of a transformation of n real variables does not necessarily imply that the transformation fails to be one-to-one. Focal points are characterized in a similar manner. (Received October 10, 1932.)

261. Dr. C. B. Morrey (National Research Fellow): *A class of representations of manifolds.*

The representation $x^i=x^i(u, v)$, $i=1, \cdots, N$, of a surface S is said to be of class L if the $x^i(u, v)$ are absolutely continuous in the sense of Tonelli and either (A) $|x_u^i|^p$ and $|x_v^i|^q$, $i=1, \cdots, N$, are summable, $1/p+1/q\leq 1$, p, $q>1$, or (B) $|x_u^i|<M$ and $|x_v^i|$ summable or $|x_v^i|<M$ and $|x_u^i|$ summable, $i=1, \cdots, N$. It is first shown that the Lebesgue area, $L(S)$, of a surface S is given by the usual integral formula. Green's formula for space and Stokes's formula are extended to situations where the surfaces involved are of class L and the function $P(x, y, z)$ is very general. Many of these results are extended to n-dimensional manifolds. If a representation is of class L and $E=G$, $F=0$ almost everywhere, it is said to be generalized conformal. It is shown that if the generalized conformal representations $x^i=x_n^i(u, v)$ of S converge in the mean (i.e., the functions do) to the continuous representation $x^i=x^i(u, v)$ of S, and $L(S_n)$ approaches $L(S)$, then the limit representation is generalized conformal. (Received October 10, 1932.)

262. Mr. J. G. Deutsch: *Partial derivates.*

In this paper we show that a classification of the partial derivates of measurable functions of two variables, analogous to the Denjoy-Young-Saks classification of the derivates of functions of one variable, is not, in general, possible. This contradicts certain theorems given by G. C. Young (Proceedings of the London Mathematical Society, (2), vol. 20 (1921–22), pp. 182–88). (Received October 6, 1932.)

263. Mr. J. G. Deutsch: *Functions having the property* (N).

It is known that, if a continuous function has the Lusin property (N) on an interval, then that interval contains a set at which the function has a derivative and on which is assumed almost every value assumed on the interval. We show further that this set can be so taken as to have almost every value assumed on the interval assumed the same number of times on the set. This added fact provides a vehicle for obtaining another proof of Mlle. Bary's theorem on the absolute continuity of functions having the property (N). (Received October 6, 1932.)

264. Dr. J. J. Gergen and Dr. S. B. Littauer: *Continuity and summability for double Fourier series.*

Let $f(u, v)$ be an even-even function, integrable in the Lebesgue sense over the square $(0, 0; \pi, \pi)$, and doubly periodic with period 2π in each variable. Let $\sigma_{m,n}^{a,b}$ be the (m, n)-Cesàro mean of order (a, b) of the Fourier series of f at the origin. Let $f^{\alpha,\beta}(x, y)=[\alpha\beta/(x^\alpha y^\beta)]\int_0^x(x-u)^{\alpha-1}du\int_0^y(y-v)^{\beta-1}fdv$ be the mean

of order (α, β) of f with respect to the origin. The first object of this paper is to study the relations between the limiting processes lim $\sigma_{m,n}^{a,b}$ as $(m, n)\to\infty$ and lim $f^{\alpha,\beta}(x, y)$ as $(x, y)\to(+0, +0)$. By applying Wiener's method of quasi-Tauberian theorems we obtain, for a certain general class of functions, results analogous to those obtained by Wiener in sharpening the familiar theorem of Hardy and Littlewood on continuity and summability of simple Fourier series. Incidentally, we show that under certain conditions the Riesz and Cesàro methods of summability for double series are equivalent. We also obtain results in connection with the iterated limiting processes lim lim $\sigma_{m,n}^{a,b}$ as, first, $m\to\infty$, and then $n\to\infty$, and lim lim $f^{\alpha,\beta}(x, y)$ as, first, $x\to+0$, and then $y\to+0$. (Received October 8, 1932.)

265. Professor E. V. Huntington: *A set of independent postulates for Principia Mathematica.*

Given the following primitive ideas: a class K ["propositions"]; a subclass T ["true" propositions]; a binary operation, $a+b$ ["a or b"]; a unary operation, a' ["not a"]. Any system $(K, T, +, ')$ which satisfies the following seven postulates may be called a "*Principia* system." (1) If a and b are in K, then $a+b$ is in K. (2) If a is in K, then a' is in K. (3) If a, b, etc., are in K, then $b'+(a+b)$ is in T. (4) If a, b, etc., are in K, then $(a+b)'+(b+a)$ is in T. (5) If a, b, c, etc., are in K, then $(b'+c)'+[(a+b)'+(a+c)]$ is in T. (6) If $a+b$ is in T, then at least one of the elements a and b is in T. (7) If a' is in T, then a is not in T. Here Postulates 1–5 correspond precisely to "formal," and 6–7 to "informal" statements in the *Principia*, but 6 and 7 are not deducible from the formal part of the *Principia*. All the propositions, "formal" and "informal," in Section A of the *Principia* are deducible from Postulates 1–7. (Received October 6, 1932.)

266. Dr. E. J. McShane (National Research Fellow): *Integrals over surfaces in parametric form.*

The author has previously (Annals of Mathematics, vol. 32, p. 460) investigated the semi-continuity of double integrals of the calculus of variations taken over surfaces $x=x(u, v)$, $y=y(u, v)$, $z=z(u, v)$, where the functions $x(u, v)$, etc., are Lipschitzian. In the present paper the theorems previously obtained are extended to a wider class of surfaces, including, e.g., surfaces for which $x(u, v)$, etc., are absolutely continuous in the sense of Tonelli and the partial derivatives $\partial x/\partial u$, etc., are summable together with their squares. For the same wider class of surfaces it is shown that the area (in the sense of Lebesgue) is given by the classical double integral. (Received September 20, 1932.)

267. Dr. E. J. McShane (National Research Fellow): *Parametrizations of saddle surfaces, with application to the problem of Plateau.*

If the surface S: $x=\bar{x}(u, v)$, $y=\bar{y}(u, v)$, $z=\bar{z}(u, v)$ is bounded by a Jordan curve and has finite area, and the functions $\bar{x}(u, v)$, etc., are monotonic in the

sense of Lebesgue, then there exists a representation $x=x(u, v)$, $y=y(u, v)$, $z=z(u, v)$, $u^2+v^2\leqq 1$, of the surface S satisfying the following conditions: (1) the functions $x(u, v)$, etc., are absolutely continuous in the sense of Tonelli; (2) the partial derivatives $\partial x/\partial u$, etc., are summable together with their squares; (3) for almost all points (u, v) we have $E=G$, $F=0$, where E, F, G have their usual meanings. This theorem, together with the previously established result that (1) and (2) ensure that the area is given by the classical double integral, leads readily to a solution of the problem of Plateau. (Received September 20, 1932.)

268. Dr. W. J. Trjitzinsky: *A property of indefinitely differentiable classes.*

Let classes C of functions $f(x)$, indefinitely differentiable on an interval (a, b), be specified by the law of increase of the absolute values of the derivatives. The author proves that no non quasi-analytic classes C exist whose members are uniquely determined by the values of the derivatives (of all orders) at $n(\geqq 2)$ points of (a, b). (Received October 6, 1932.)

269. Professor J. L. Walsh: *On the approximation of analytic functions by rational functions of best approximation.*

Let the function $f(z)$ be analytic in the interior of the Jordan region C and continuous in the corresponding closed region. Let $R_{mn}(z)$ be a (necessarily existent) function of the form $(a_0z^m+a_1z^{m-1}+\cdots+a_m)/(b_0z^n+b_1z^{n-1}+\cdots+b_n)$, $b_0z^n+b_1z^{n-1}+\cdots+b_n\not\equiv 0$, of best approximation in the sense of Tchebycheff to $f(z)$ in C. Then any infinite sequence of the $R_{mn}(z)$, for which the first subscript has no lower limit less than the number of zeros of $f(z)$ interior to C, converges to $f(z)$ uniformly in the closed region C. Overconvergence may take place. The doubly infinite array of the $R_{mn}(z)$ is in many respects analogous to the Padé table. (Received October 6, 1932.)

270. Professor W. A. Wilson: *On topological characterizations of the two-dimensional simplex.*

Let Z be a locally connected compact metric continuum containing a simple closed curve J and satisfying the following conditions: (a) if ab is a simple arc in Z such that $(ab)\cdot J=a+b$, $Z-ab$ has exactly two components and these contain the respective components of $J-(a+b)$; (b) if α and β are disjoint open arcs of J and K is a closed sub-set of $Z-(\alpha+\beta)$ of which no component meets both components of $J-(\alpha+\beta)$, there is a simple arc ab in $Z-K$, from a point a in α to a point b in β, which has no other points on J. It is shown that the methods used by H. Whitney in his combinatorial characterization of the two-dimensional simplex can be employed with little change to prove that the continuum Z defined above is the homeomorphic image of a two-dimensional simplex. (Received October 3, 1932.)

271. Professor Edward Kasner: *Families of heat surfaces.*

This paper considers the geometry of Fourier's equation for the conduction of heat in three dimensions, extending some of the two-dimensional results of

earlier papers (see Proceedings of the National Academy of Sciences, vol. 18 (1932), pp. 475–480). A heat family of surfaces contains in general ∞^2 surfaces, one for each time and each temperature. There are only three types of degenerate families, for which there are merely ∞^1 surfaces, and these are defined by the equations of Laplace, Poisson, and Helmholtz or Lamé. No doubly-infinite family can be composed of planes. (Received October 10, 1932.)

272. Professor Harold Hotelling: *Analysis of a multivariate complex into principal components.*

The problem of extracting and measuring the common elements in an aggregate of statistical variables, and particularly of determining whether the number of independent common elements can be taken as less than the number of observed variables, has received considerable attention from E. B. Wilson, C. Spearman, T. L. Kelley, E. L. Thorndike, and other mathematical psychologists. The same logical and mathematical considerations apply in other fields also, as Wilson and Shewhart have pointed out. The present paper is concerned with the selection of a system of independent components of which the first, among all linear functions, makes the maximum contribution to the total variance, and the others, in sequence, make the maximum contributions to the residual variances in turn. This is shown to be geometrically equivalent to the selection of the principal axes of an n-dimensional ellipsoid in the space of the scatter diagram. An iterative process is offered which reduces the otherwise enormous calculation necessary for the application of this idea to a moderate amount, even for numerous variables. The sampling problems associated with this type of analysis are examined, and certain approximate solutions found, which are capable of distinguishing between cases in which Spearman's single-general-factor theory is relevant and those in which it is not. The problem of errors of measurement is also considered. (Received October 8, 1932.)

272. Dr. Stanislas Saks: *On some functionals.*

The author gives a new proof and various extensions of the following theorem of Hahn: If $\{f_n(x)\}$ is a sequence of integrable functions and if the sequence of the integrals $\int_E f_n(x)dx$ converges for every measurable set E, then the indefinite integrals $F_n(t)=\int_a^t f_n(x)dx$ are equally continuous in every finite interval. (Received September 7, 1932.)

273. Professor Virgil Snyder: *On a series of Cremona involutions defined by a pencil of ruled surfaces.*

Consider a pencil of surfaces $|F_n|$ through a rational curve r to multiplicity $n-2$. Let the points M of r and the surfaces of the pencil $|F|$ be in $(1, k)$ correspondence. A point P fixes a surface F of the pencil and this in turn a point M on r. The line PM meets F_p in one residual point P'. The relation between P, P' is an involutorial birational transformation of space. The case in which r is a straight line has been solved by Carroll (American Journal, vol. 54 (1932)). That in which r is a proper curve and the residual base simple and irreducible has been considered by Black (Transactions of this Society, vol. 34

(1932)). The latter includes that of the (ruled) quartics through a double space cubic curve r. The present paper discusses all possible cases in which every surface of the pencil is ruled. The interest lies in the rôle played by the fundamental lines of the second kind, and the contact conditions along the directrix curve. The new transformations include a number of well known types, but furnish infinitely many new ones in which m (number of generators through a point of p), $n-2$, k can each take any positive integral value. (Received October 19, 1932.)

274. Dr. A. T. Craig: *Distributions of functions of middle items.*

In the present paper, laws of probability are determined in accord with which various functions of the middle n items of samples consisting of $2k+n$ items are distributed. Among the functions considered are the arithmetic mean, the average of the extreme values, and the range. An experimental study of certain aspects of this problem has been made by E. S. Pearson (Biometrika, vol. 20A, 1928). (Received November 3, 1932.)

275. Mr. F. A. Brandner: *A test of the significance of the difference of the correlation coefficients of normal bivariate samples.*

By using the method of likelihood, the author has procured a criterion to test the significance of the difference of the correlation coefficients of two normal bivariate samples. The ratio of the chance of procuring the samples with the added hypothesis that the correlation coefficients are identical, to the chance of procuring them with only the condition that they come from normal bivariate populations, is set up as a test. This ratio proves to be a function of R. A. Fisher's well known expression z_1-z_2. Thus the criterion z_1-z_2 is shown to be a logical test to use in case nothing is known as to the values of the remaining parameters. (Received October 31, 1932.)

276. Professor E. S. Allen: *A generalized definition of probability.*

In view of the fact that there is never any certainty that a frequency sequence encountered in nature converges, it is natural to ask what can be regarded as probability in case of divergence. The present paper shows that the sequence itself satisfies Reichenbach's axioms of probability with one slight alteration; and investigates the modifications required in the resulting theory. (Received November 4, 1932.)

277. Mr. Deane Montgomery: *Sections of point sets.*

This paper considers the problem of characterizing a plane point set, E, in terms of properties of subsets of E, called horizontal and vertical sections of E, which are the products of E and given horizontal and vertical lines. The theory is developed for the more general case in which E is a subset of the combinatorial product of metric spaces. Theorems are given concerning the Borel classification of E when its vertical sections are a certain type of closed or open set and its horizontal sections are F or O of class α. The category of E is re-

lated to the categories of its sections by theorems additional to those of Kuratowski and Ulam (Fundamenta Mathematicae, vol. 19, p. 246). Certain applications are made to functions of two variables continuous in each of them. (Received November 3, 1932.)

278. Professor H. R. Pyle: *The conditions for conformality in the elliptic and hyperbolic geometries.*

This paper develops the differential equations which are the necessary and sufficient conditions for conformality in the elliptic and hyperbolic plane geometries. These equations contain the Cauchy-Riemann equations as the special case in which the curvature constant is infinite. They lead to a generalization of the Laplace equation which reduces to that equation in the euclidean case. The function $W=\phi+i\psi$, conformal in the non-euclidean plane, is shown to be the reciprocal of a harmonic polygenic function, although it is not itself harmonic. In order to find the above conditions, a transformation which carries points from the non-euclidean plane into corresponding points in the euclidean plane is used. Standard forms of functions conformal in the non-euclidean plane are found in terms of functions conformal in the euclidean plane. (Received October 26, 1932.)

279. Professor M. H. Ingraham: *On the reduction of a matrix to its rational canonical form.*

Let A be an nxn matrix with coefficients in a field F. Let $L=L(\xi_1 \cdots \xi_p)$ be the linear set consisting of the totality of vectors of the form $\sum_1^p f_i(A)\xi_i$ where the f's are polynomials with coefficients in F and the ξ's are $nx1$ matrices (vectors) with elements in F. If η is a vector, we say that $g(A)\eta \equiv 0 \bmod L$ if $g(A)\eta$ is in L. It is shown that for each vector η there is a function not identically 0 of minimum degree effective as g. This function divides all other polynomials effective as g. There is an η which maximizes the degree of the corresponding minimum function g, and the minimum function g_1, corresponding to any other vector η_1, divides g. From these considerations one readily develops the reduction of A to a canonical form by transformations in F. (Received November 2, 1932.)

280. Professor H. T. Davis: *A technique for the study of the interaction of economic series.*

The present paper, the product of computations made by the Cowles Commission for Research in Economics, sets forth a technique which is applicable to the study of the interaction effects of several economic series. Let us assume that the effect of three series, (y_i), (y_i'), and (y_i''), upon one another is desired. A relationship exhibited by correlation coefficients of magnitude greater that .5 will be assumed to exist between the three series. First and second derivative curves, (Δy_i), $(\Delta^2 y_i)$, are constructed for each series by first computing the differences and then smoothing these by means of moving averages. The three series, the original and the two derivative series, are thus reduced to approximately the same degree of smoothness. The nine series obtained in this manner are then correlated into three difference equations by correlating the second

derivative curves in turn with the other eight. This system of equations is then studied for its oscillations and the results obtained are interpreted in the light of the periodogram for each series. The method has been applied to the three series: Dow Jones averages, pig iron production, and Bradstreet commodity price index, for the pre-war data. (Received November 3, 1932.)

281. Professor David Moskovitz: *Certain irregular non-homogeneous linear difference equations.*

This paper treats the non-homogeneous linear difference equation $\sum_{k=0}^{n} a_k(x)g(x+n-k)=b(x)$ in which $a_k(x)$ and $b(x)$ are rational functions, or only of a rational character at infinity, and for which the associated homogeneous equation does not belong to the so-called "regular" case, its characteristic equation being permitted to have zero or infinite roots. Under various assumptions analytic solutions are obtained and their asymptotic forms studied. Two methods are employed, one analogous to that used by K. P. Williams (Transactions of this Society, vol. 14, p. 209) in studying the non-homogeneous equation whose associated homogeneous equation is regular; this method extends the results obtained by Williams to the class of equations here studied, and certain of Williams' results are amplified. The second method transforms the non-homogeneous equation to a homogeneous equation of one higher order, which is shown to belong to the same irregular case as the homogeneous equation associated with the non-homogeneous equation under study. The results of C. R. Adams (Transactions of this Society, vol. 30, p. 507) concerning the solutions of homogeneous difference equations in this irregular case may be applied at once. (Received October 31, 1932.)

282. Dr. M. C. Gray: *Mutual impedance of long grounded wires when the conductivity of the earth varies exponentially with depth.*

Assuming that the conductivity of the earth increases exponentially with depth $(-z)$ according to the formula $\gamma=\gamma_0 e^{-z/c}$, where γ_0 is the conductivity at the surface $z=0$, this paper gives a formula for the mutual impedance, per unit length, of two long parallel wires, grounded at their end-points. The earth is supposed flat, semi-infinite in extent, and the inductivities of both earth and air to be equal to that of free space ν. The frequency, $\omega/(2\pi)$, is assumed to be sufficiently small to allow all displacement currents to be neglected. The wires are at heights h and H above the earth's surface, and the horizontal separation between their vertical planes is y. The propagation constant at the surface is $\Gamma_0=(i\omega\nu\gamma_0)^{1/2}$. For $c=\pm\infty$ this formula reduces to that given by J. R. Carson (Bell System Technical Journal, vol. 5, pp. 539–554) for a uniformly conducting earth, while if c approaches zero through negative values it agrees with that given by O. Mayr (Elektrotechnische Zeitschrift, vol. 46, pp. 1352–1355) for an earth consisting of a conducting layer at the surface only. (Received November 1, 1932.)

283. Professor D. L. Holl: *The deflection of a rectangular plate, supported at two opposite edges, due to a point load.*

A. E. H. Love (Proceedings of the Royal Society, London, vol. 118 (1928)) solved this problem for a *centrally* loaded plate by conformally mapping the rectangle upon a unit circle and determining five biharmonic functions satisfying the necessary boundary conditions. The functions are represented by infinite series and in some case the constants determined are expressed by infinite sums. In this paper the solution is effected by dividing the plate into three rectangles, one of which is uniformly loaded on a rectangular portion of it. For these three sections, suitable solutions of $\nabla^4 w = f(x, y)$ are chosen so as to satisfy the edge conditions and the necessary continuity properties at the adjacent sections. The solutions are in series form with closed expressions for the constants. By enlarging the loaded area or by shrinking it to remain a finite point load, the solutions include the cases of uniformly loaded plates or infinite plate strips as well as the rectangular finite plate or infinite strip with a concentrated load at any point. (Received October 31, 1932.)

284. Professor W. C. Brenke: *On the summability and generalized sum of the series of Legendre polynomials* $\sum n^p X_n(x)$.

In this paper it is shown that, for positive integral values of p, the series $\sum n^p X_n(x)$ is summable by the Hölder mean value process of order p, that is, summable (H, p), for the range $-1 < x < 1$. It is summable $(H, p+1)$ for this range and including $x = -1$. The sum is obtained in closed form. For $x = -1$ this gives the sum of the oscillating series $\sum (-1)^n n^p$. (Received November 1, 1932.)

285. Professor Dunham Jackson: *Problems of approximation with integral auxiliary conditions.*

This note discusses the modification to which a problem of closest approximation according to the criterion of least mth powers is subjected if it is required by way of auxiliary condition that certain definite integrals involving the approximating function agree exactly in value with the corresponding integrals in terms of the function to be approximated. It is shown that hypotheses which would naturally be formulated to insure convergence in the absence of the auxiliary conditions remain adequate when the additional conditions are imposed. (Received October 21, 1932.

286. Professor J. V. Atanasoff: *Solution of Dirac's equation without specialization of the operators.*

Sauter, in two papers in the Zeitschrift für Physik (vol. 63, p. 803; vol. 64, p. 295), has given a simple direct method for reducing Dirac's equation to a system of differential equations in ordinary numbers without specializing the Dirac operators. However, Sauter's solution for ψ possesses a more complicated structure than the situation demands. As a result an additional hypothesis is needed in forming the quantities quadratic in the ψ's. In the present paper the ψ's are assumed to depend in a linear way upon the characteristic operands of the Dirac operators. In this way results equivalent to Sauter's are obtained without the need of his hypothesis. (Received November, 2, 1932.)

287. Mr. C. P. Wells: *Separation of partial differential equations in two unknowns.*

A definition of a partial differential equation that is separable in specified coordinates is given. This definition is first applied to the study of the coordinates that separate Laplace's equation in two independent variables. The principal result here is that if u, v separate this equation, there exist functions U of u alone and V of v alone which satisfy the Cauchy-Riemann equations. This definition is also applied to a self-adjoint partial differential equation of the elliptic type, in the study of which extended Cauchy-Riemann equations are employed. These equations have much the same relation to this problem that the original Cauchy-Riemann equations have to the separation of Laplace's equation but an additional condition must be imposed before the conditions are sufficient to insure the separation of the equation. (Received November 2, 1932.)

288. Mr. Fred Robertson: *The fractional differentiation operator.*

The function of x defined by $\int_0^x \{s^{n-1}e^{-sz}/\Gamma(n)\}ds$, $(n>0)$, where z is the operator d/dx arises in the calculus of operators. The author considers some properties of these functions of x when n assumes negative values. The fractional derivative ${}_0D_x^{\alpha}u(x)$ is expressed in the operational form $\{[x^{-\alpha}/\Gamma(-\alpha)] \cdot \sum_{r=0}^{\infty}(-1)^{r+1}[x^r z^r/(\alpha-r)r!]\} \to u(x)$ where α replaces $-n$ and is not an integer. This formula is very convenient for computing fractional derivatives. The theory of these fractional derivatives is developed by means of a second order partial differential equation containing a parameter α. (Received November 5, 1932.)

289. Mr. R. H. Cameron: *Almost periodic transformations.*

The aim of this paper is to define and study a class of one-to-one uniformly continuous transformations of the points of a closed set in a complete metric space. The definition of these transformations, called almost periodic transformations, is analogous to Bohr's definition of an almost periodic function and Walther's definition of an almost periodic sequence. On the basis of this definition, certain fundamental theorems are proved: for instance, *the product of any two permutable almost periodic transformations is almost periodic.* Next, the notion of a power of an almost periodic transformation is very much extended, and the properties of these new powers are shown to be similar to those of ordinary powers defined by iteration. Finally, a special case of an almost periodic transformation called a mono-basal transformation is defined, and it is shown that every almost periodic transformation can be expressed as a finite or infinite product of mono-basal transformations. (Received October 28, 1932.)

290. Mr. Joseph Lev: *The effects of linear transformations on the divergence of bounded sequences and functions.*

This paper is a new approach to a problem previously studied by Hurwitz and Knopp. The *limit circle* of a bounded sequence of complex terms is defined

as the circle of least radius which contains within or on its boundary the limit points of the sequence. If the transformation is given by $y(t)=\sum_{i=1}^{\infty}K_i(t)X_i$, where the sequence $[x_i]$ is bounded, and the coefficients $K_i(t)$ are suitably restricted, then the limit points of $y(t)$, as t approaches t_0, lie in a circle whose center and radius are expressed linearly in terms of the center and radius of the limit circle of the sequence. The new concept permits a study of a wider class of transformations than those used by earlier writers. Similar results are obtained when linear transformations are applied to the complex function $f(x)$ of the real variable x, and the limit circle of $f(x)$ at a point is defined as in the case of sequences. Applications are made to the study of Cauchy products and convergence factors in the theory of infinite series. (Received October 27, 1932.)

291. Professor N. H. McCoy: *On quasi-commutative matrices.*

Two square matrices x and y, of order n, may be said to be *quasi-commutative* if $xy-yx$ is commutative with both x and y. It is proved that if z is a given matrix of order n, a necessary and sufficient condition that there exist quasi-commutative matrices x and y such that $xy-yx=z$, is that the elementary divisors of z be of the form $\lambda^{n_1}, \lambda^{n_2}, \cdots, \lambda^{n_k}$, with n_i-n_{i+1} either zero or unity and $n_k=1$. Several well known properties of commutative matrices are shown to hold also for quasi-commutative matrices. In particular, if $f(x, y)$ is a scalar polynomial in x and y, then the roots of $f(x, y)$ are of the form $f(\alpha_i, \beta_j)$, where α_i is a root of x and β_j a root of y. Some additional properties of quasi-commutative matrices are obtained by means of known theorems in the algebra of quantum mechanics, as the ring of polynomials in x and y is homeomorphic to the ring of polynomials in the infinite matrices (or operators) p and q of quantum mechanics. (Received November 3, 1932.)

292. Professor J. L. Walsh: *On approximation by non-vanishing analytic functions.*

Let the functions $\phi(z)$ be analytic and univalent (i.e., schlicht) interior to the Jordan region C, and let $\phi(z)$ have at least one (hence precisely one) zero interior to C. Let $w=\delta$ be the boundary point (or one of the boundary points) nearest the origin of the region into which C is transformed by $w=\phi(z)$. Then for the particular function $f(z)=\phi(z)-\delta$, which is analytic and different from zero interior to C, we have $|f(z)-\phi(z)|=|\delta|$, z in C. If any function $F(z)$ analytic interior to C is such that we have $|F(z)-\phi(z)|\leqq\delta'<|\delta|$, z in C, then $F(z)$ has precisely one zero interior to C. (Received October 26, 1932.)

293. Dr. G. T. Whyburn: *On the existence of totally imperfect and punctiform connected subsets in a given continuum.*

A set containing no compact perfect subset [continuum] is totally imperfect [punctiform]. Sierpinski has shown that in any euclidean space $E_n(n>1)$ the complement of any totally imperfect set is connected. In this paper it is shown that Sierpinski's theorem may be so generalized as to read as follows: In any locally compact continuum M, the complement of every totally imperfect set augmented by the set of all local separating points of M is connected. Using this proposition, we obtain the following. (1) Let M be a locally

compact continuum and let L be the set of all local separating points of M; then in order for M to contain, for each $x \in M$, a totally imperfect connected set containing x it is necessary and sufficient that L be countable. (2) Also if M is locally connected, it will contain, for each $x \in M$, a punctiform connected set containing x if and only if L is punctiform. (3) In order that the hereditarily locally connected continuum H contain no punctiform connected subset, either of the following conditions is necessary and sufficient: (a) that the set of local separating points of no sub-continuum of H be punctiform, (b) that every cyclicly connected sub-continuum of H have a free arc. (Received November 3, 1932.)

294. Dr. G. T. Whyburn: *Characterizations of certain curves by continuous functions defined upon them.*

Čech has shown (Fundamenta Mathematicae, vol. 17) that any compact continuum M upon which there can be defined a real, continuous function which is not constant on any infinite subset of M is a particular kind of regular curve. Mazurkiewicz (Fundamenta Mathematicae, vol. 18) has given necessary and sufficient conditions that a given acyclic locally connected continuum (dendrite) have the property of admitting such a function to be defined on it. In the present paper it is shown that, by varying the restrictions on the function, regular, rational, and 1-dimensional curves (Menger-Urysohn sense) may be characterized. Indeed it is shown that in order for a compact continuum M to be (1) a regular curve, (2) a rational curve, (3) a 1-dimensional curve, it is necessary and sufficient that there exist on M a real, continuous function which is not constant on any subcontinuum of M and such that there exists an everywhere dense set of its values each of which it takes only (1) a finite number, (2) a countable number, (3) any number, respectively, of times. (Received November 3, 1932.)

295. Professor A. D. Campbell: *Plane quartic curves in the Galois fields of order 2^n.*

Plane quartic curves in the Galois fields of order 2^n have many peculiarities. Every point has a cuspidal polar cubic with respect to such a quartic. The tricuspidal quartic does not exist. There are in general only seven bitangents to a quartic, which arrange themselves respectively as the six sides of a complete quadrangle, and the line containing the diagonal points of this quadrangle. Also there are quartics having each four tangents at undulations, and three bitangents that form the same configuration as the above mentioned seven bitangents. The Plücker equations for plane algebraic curves do not hold for a quartic. We find even a quartic with a cusp P and with every line through a certain other point proving to be a bitangent to the curve. Finally, we note that only a very few types of quartics can be first polars of points with respect to a quintic. (Received November 5, 1932.)

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OFFICIAL COMMUNICATIONS

Meetings of the Society have been fixed at the following times and places:

AMES, IOWA, November 25-26, 1932.

Abstracts must be in the hands of Associate Secretary M. H. Ingraham, University of Wisconsin, Madison, Wisconsin, not later than November 3. Abstracts received by that date will appear in the November issue of this BULLETIN. By invitation of the program committee, Professor E. B. Stouffer will speak on *Some canonical forms and their associated geometries*, and Professor H. W. March will speak on *Applications of the theory of elasticity to wood, a material of anisotropic structure.*

ATLANTIC CITY AND PRINCETON, NEW JERSEY, ANNUAL MEETING, December 27-30, 1932.

Abstracts must be in the hands of the Secretary of the Society, 501 West 116th St., New York City, not later than November 30, 1932. Abstracts received by November 7 will appear in the November issue of the BULLETIN. There will be a joint session of Section A and The Mathematical Association, at which time Dean L. P. Eisenhart will deliver his retiring presidential address on the topic *Some recent developments in differential geometry*, and Professor E. R. Hedrick will give his retiring address as Chairman of Section A of the A.A.A.S., his topic being *Tendencies in the logic of mathematics.* There will be a joint session with the Econometric Society and Sections A and K. The tenth Josiah Willard Gibbs Lecture will be delivered by Professor R. C. Tolman of the California Institute of Technology on the topic *Thermodynamics and relativity.* On the afternoon of December 30, at Princeton University, there will be a symposium on *Applications of the operational calculus to mechanics*, at which time Professor J. von Neumann of Princeton University and the University of Berlin will be the main speaker and Professor G. D. Birkhoff will lead the discussion.

NEW YORK CITY, February 25, 1932.

Abstracts must be in the hands of the Secretary of the Society, 501 West 116th St., New York City, not later than February 4, 1932. Abstracts received by January 7 will appear in the January issue of the BULLETIN.

R. G. D. RICHARDSON, *Secretary of the Society.*

Articles for insertion in the BULLETIN should be addressed to E. R. HEDRICK, Editor of the BULLETIN, University of California at Los Angeles. Reviews should be sent to W. R. LONGLEY, Yale University, New Haven, Conn. Notes should be sent to H. W. KUHN, Ohio State University, Columbus, Ohio.

Subscriptions to the BULLETIN, orders for back numbers, and inquiries in regard to non-delivery of current numbers should be addressed to the American Mathematical Society, 450-459 Ahnaip St., Menasha, Wis., or 501 West 116th St., New York.

The initiation fees and the annual dues of members of the Society (see this BULLETIN, p. 322, May, 1930; and the List of Officers and Members, October, 1930, p. 58), are payable to the Treasurer of the Society, Professor G. W. Mullins, 501 West 116th St., New York City.

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